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THE SUPERVISION OF ARITHMETIC

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PREFACE

THIS is not a teacher's handbook for the teaching of arithmetic; it does not contain a detailed description of the methods and devices teachers should employ in teaching arithmetic, nor is it intended to be in any sense a critical analysis of the many scholarly articles that have appeared within recent years bearing upon the subject. It is frankly the result of a number of surveys and investigations conducted by the authors themselves or their students of certain problems relating to the supervision of arithmetic. It is not maintained that every problem relating to the supervision of arithmetic has been solved.

This book gives the inquiring and progressive supervisor certain criteria for judging his course of study in arithmetic and in addition it gives him certain tests for measuring the attainments of his pupils. Certainly its greatest value lies in the fact that it raises to consciousness and outlines more clearly than heretofore a number of problems connected with the supervision of arithmetic.

THE AUTHORS.

CONTENTS

THE SUPERVISION OF ARITHMETIC

CHAPTER I

SUBJECT MATTER OF ARITHMETIC

Current Criticisms

Most critics of the public schools are inclined to maintain the thesis that the work in arithmetic of to-day is less satisfactory than the work which was done in this subject at an earlier period. They say that the old-fashioned teacher spent more time in drill, and less time in rationalization or explanation. The critic of to-day is disposed to call attention to the fact that certain factors and processes which are being taught in arithmetic are of questionable value as far as the practical demands of the day are concerned. The usual illustrations cited are: tables — furlongs, apothecaries' weights, foreign money, folding paper, longitude and time, and the like. Much criticism is also heard against the teaching of true discount, cube root, partnership, unreal fractions, and alligation.

Again, it is not infrequently stated that more attention should be given both in the fundamentals and in the applications of arithmetic to social conditions connected with the problems of saving, banking, borrowing, building and loans, investments, bonds and stocks, taxes, levies, public expenditures, and insurance.

Both of these criticisms contain certain elements of validity. Not only is it difficult for the school to discriminate between the essential elements of the experience of the past, but it has no well-defined standards for discriminating values of the present. The immaturity, lack of experience, and inadequacy of training of teachers have resulted in a more or less slavish dependence upon the textbook. Indeed, in many instances the textbook serves as the entire course of study, and furnishes the only criterion of method.

The expense of changing textbooks and the difficulties attending their adoption have contributed to a certain amount of conservatism and inflexibility in their use. Again, the teacher's immaturity and lack of experience have tended toward the exclusive use of the problems and examples found in the textbook. In other words, there has been too little disposition on the part of teachers to supplement intelligently and to make concrete the prescribed material of the textbook. The result of these tendencies is that the material has been more or less formal and artificial.

Changing Conceptions of Arithmetical Values

Despite the fact that men and women are to-day investing in stocks, bonds, insurance, buildings, and loans, the school tends to give but slight attention to these forms of applied arithmetic, but clings to those applications of arithmetic which relate to older forms of investment, such as surveying, the buying and selling of land and other tangible property such as houses and stores. Although it is not infrequently stated by school teachers that stocks, bonds, and insurance should not be taught, there is, on the contrary, much evidence to indicate that we should spend much more time in presenting these phases of arithmetic than we have heretofore expended. However, if we expect pupils to profit by the school's attempts to teach these new fields, it is necessary that they be taught by teachers who thoroughly understand these forms of quantitative experience.

Other forms of mathematical experience with which pupils should be familiarized are public expenditures, tax levies, and assessments. It is imperative that the good citizen of the future understand fully the arithmetic involved in the assessment and levying of taxes, the issuance of public bonds, and public expenditures.

Many of the so-called "practical" courses which have been introduced in recent years have failed to emphasize

the quantitative relationships of our social and economic life. Consequently children have been fed stereotyped problems. The child who studies arithmetic at the present time has a right to be familiarized with the quantitative problems of the day. The failure of many teachers to understand clearly the arithmetical applications in the various fields of social activity has been responsible for much of the criticism directed at the product of our schools by business men. Teachers of arithmetic have been satisfied to have the children "work the problems" in the book. The children have been satisfied to find the "answers." Neither teachers nor pupils have been forced to go to the bottom of the subject in such a way as to understand the issues involved.

Recently an editorial writer of one of the leading American periodicals made the statement that arithmetic properly taught would make it impossible to exploit wild-cat mining schemes promising to pay fabulous returns. The writer said further that proper arithmetic teaching should include a thorough discussion and interpretation of sound business principles.

Two Phases of Arithmetic

Experimental researches in modern educational psychology have tended to divide arithmetic into two well-defined fields: one, that of drill wherein a child is

taught to add, subtract, multiply, and divide skillfully whole numbers and fractions; the other, wherein a child is taught to appreciate arithmetical situations, to realize differences in value, and to understand the principles operative in business to-day. A clearer recognition of this differentiation would mean an increased emphasis on the drill operations of arithmetic, as well as a clear-cut descriptive treatment of the broader applications of these habits of arithmetical procedure in concrete social and economic situations.

That there is a demand for greater skill on the part of children in ability to add, subtract, multiply, and divide is indubitably true. There is a demand for greater accuracy and speed. The supervisor will find it worth while to devise plans for an increased expenditure of energy on the part of the teacher in this direction. We may, no doubt, expect to see an increased amount of carefully directed drill work in connection with this phase of arithmetic.

On the other hand much of that part of the subject matter of arithmetic which is taught for the sake of giving the child an opportunity to understand arithmetical relationships in public and private life should be treated by detailed discussions setting forth the factors involved. There should be whole recitation periods during which attention should be focused entirely upon a discussion of material of an explanatory

character, and upon a critical analysis of those social relationships which involve arithmetic. For example, a teacher might spend several days in teaching the issues involved in life insurance, in taxation, or in public school expenditures.

The Supervisory Problem

If the results indicated in the foregoing discussion are to be attained, it will be necessary for the supervisor to assure himself that his teachers are actually familiar with the issues involved in the formation of arithmetical habits on the one hand, and with the underlying mathematical principles involved in the interpretation of business situations on the other.

While it is true that it is a more or less difficult thing to do, yet it is by no means impossible for the supervisor to develop an inquiring type of mind on the part of teachers, which will result in an understanding of the principles involved in social and economic arithmetical situations. Many commercial teachers visit commercial institutions and, indeed, not a few seek employment in such institutions in order to gain a first-hand knowledge of the commercial practices involved. While it is not recommended that classroom teachers resign for the purpose of familiarizing themselves with the commercial phase of arithmetic, it is urged that they take the steps necessary to secure this knowledge.

Teachers might profitably interview bankers, insurance solicitors, public accountants, and brokers. Now and then supervisors might profitably assemble the teachers of arithmetic for a frank consideration of such important aspects of the subject.

Textbook writers might wisely devote more attention to these matters. At the present time many of the textbooks afford very meager descriptive material dealing with such activities and processes. It should be said, however, that there is already a tendency on the part of textbook writers to give more attention to these phases of the subject, and it is to be hoped that the present tendencies will be accentuated. It should be borne in mind, however, that this will not relieve the supervisor of the responsibility of utilizing every rational agency for making his teachers conscious of the fundamental character of this problem. Nor will the textbook relieve the teacher of the same necessity of drawing as much of this material as possible from outside activities.

The studies in elimination and retardation which have been carried on in recent years reveal the fact that a large proportion of pupils do not complete the elementary school course. In view of this condition it is of great importance that they be taught the arithmetical applications to social situations in an elementary way as early as possible in their school career.

It is certainly unwise to postpone all such instruction until the eighth grade.

Attitude of Superintendents

In order to ascertain something of the attitude of city superintendents toward the elimination of certain questionable subject matter and the introduction of certain other new subject matter, a questionnaire[1] was sent by the authors to seventeen hundred superintendents. Replies returned from more than eight hundred city superintendents indicate that, at the present time, there is a well-defined disposition toward the elimination of certain more or less obsolete material. This attitude is expressed quite clearly in Table I.

The meaning of this table becomes clear when read thus: — 72 per cent of the superintendents in cities of 100,000 and over favor the elimination of apothecaries' weight; 75 per cent favor the elimination of troy weight; 53 per cent of the superintendents in all cities favor the elimination of apothecaries' weight; and 42 per cent favor the elimination of troy weight.

[1] This investigation was made in connection with a report for the Committee of Economy of Time of the National Education Association. Parts of this material were published in the Proceedings of the N. E. A., the Elementary School Teacher, and the Fourteenth Year Book of the National Society for the Study of Education.

TABLE I

PER CENT OF SUPERINTENDENTS WHO FAVOR ELIMINATION OF THE TOPICS, DISTRIBUTED SO AS TO REVEAL DIFFERENCES IN ATTITUDE IN LARGE CITIES, CITIES AND COUNTIES, AND FOR ALL CITIES

Topic	Cities of 100,000 and Over	Cities and Counties	All Cities
	Per Cent	Per Cent	Per Cent
Apothecaries' weight	72	51	53
Troy weight	75	40	42
Furlong	85	70	72
Rood in square measure	13	10	20
Dram	78	19	60
Quarter in avoirdupois	81	66	68
Surveyors' tables	72	43	47
Foreign money	25	27	28
Folding paper	63	34	35
Reduction of more than two steps	25	21	22
Long measure of G. C. D.	66	33	35
Least common multiple	25	20	22
True discount	66	44	47
Cube root	56	41	46
Partnership	50	23	25
Compound proportion	72	49	52
Compound and complex fractions	28	24	26
Cases in percentage	28	18	20
Annual interest	63	40	41
Longitude and time	13	7	8
Unreal fractions	72	70	74
Alligation	88	82	85
Metric system	41	19	20
Progression	88	63	67
Aliquot parts	9	19	21

Chart I shows the per cent of superintendents who favored the "elimination" of certain topics, or the giving of "less attention" to certain topics, or were

"satisfied" with the emphasis given to the topics. The cross-hatched portion indicates the per cent of superintendents who favor the elimination of each of these subjects. The shaded portion indicates the per cent of superintendents who favor giving less attention to these subjects; and the remaining portion indicates the per cent of superintendents who are satisfied with the present condition. It should be borne in mind, however,

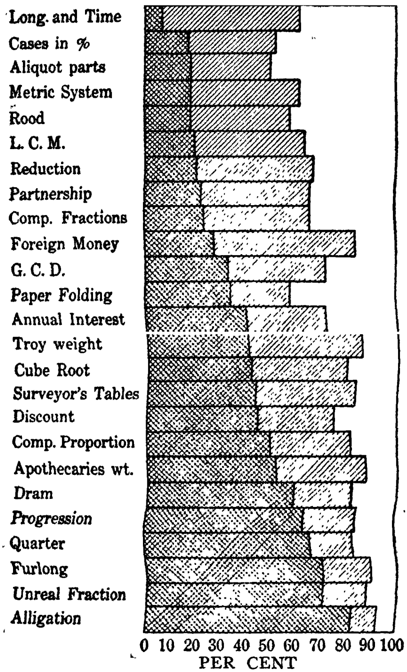

CHART I.

in interpreting this chart, that it is possible that those superintendents who are satisfied with the present degree of emphasis may have already decreased the amount of attention given to the various topics in their own schools; likewise, it is clear that the use of the word "less" in this particular is a relative matter;

we do not know what "less than" means. The response, however, is such as to convince a student that the superintendents of this country are not in favor of paying much attention to these topics.

GRAPHIC REPRESENTATION OF ELIMINATIONS

Charts II and III present new distributions of the facts found in the preceding table. The two charts show that greater variability of opinion exists among superintendents in cities of 100,000 and above than among the superintendents of the smaller cities or cities in general.

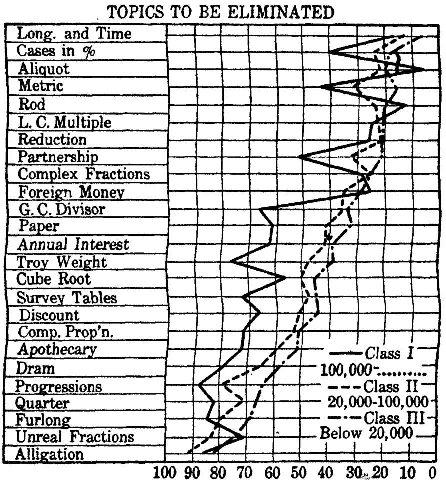

CHART II.

They can be read easily. For example, Chart II shows that 17 per cent of the superintendents in cities of 100,000 or more favor the elimination of longitude and time, 40 per cent favor the elimination of cases in percentage, 10 per cent aliquot parts, 45 per cent the

metric system, etc. Chart III may be read in the same way.

Recommendations

Because of the actual support given by the superintendents to the theoretical reasons for emphasizing material of distinct social value, it seems safe to recommend that the following subjects be given little or no attention in elementary courses of study in arithmetic, alligation, unreal fractions, furlongs, progression, dram, apothecaries' weight, compound proportion, true discount, surveyors' tables, cube root, troy weight, annual interest, paper folding, long method in greatest common divisor, foreign money, compound fractions, partnership, reduction of more than two steps, least common multiple, rod in square measure, metric system, aliquot parts, cases in percentage, longitude, and time.

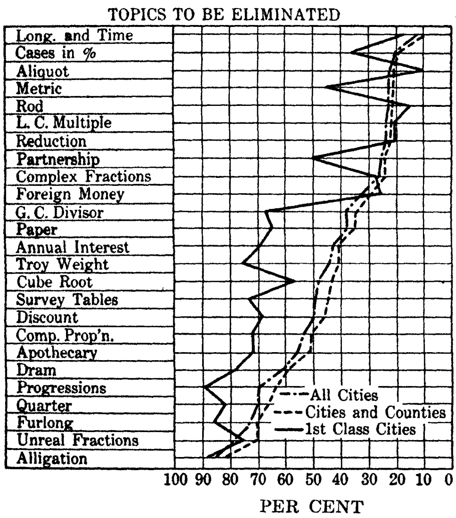

CHART III.

No doubt there are other subjects which might be added to this list. This does not necessarily mean that these topics should be left out absolutely. It does mean, however, that instruction in these fields might properly be minimized in the arithmetic of the elementary grades. Longitude and time, for ordinary purposes, might be very properly taught in connection with the general topic of railroading, wherein the different kinds of time might be explained in a purely descriptive way, so as to enable the children to understand the facts in connection with standard time throughout the different divisions of the United States. Partnership might be taught more profitably under the general head of Corporations in connection with the issuance of stocks and bonds. For the most part, however, little time should be given to the topics listed above. The fact that superintendents throughout the country are dissatisfied with conditions as they exist to-day should encourage us to believe that we are about to change our practice with regard to these topics.

Since a large proportion of the pupils do not complete the elementary school course, it is necessary that the school take cognizance of this so as to guarantee that the normal child who leaves school at an early age be proficient in the manipulation of the fundamentals of arithmetical procedure. To do this, serious attention should be given to the problem of teaching the fundamental processes effectively.

Topics Demanding Increased Emphasis

The same superintendents who were asked to express their opinions with regard to their attitude toward the subject matter indicated above, expressed themselves as overwhelmingly in favor of giving more attention to the fundamentals of addition, subtraction, multiplication, division, and fractions.

The replies of the superintendents favoring increased emphasis are presented in tabulated form in Table II.

TABLE II

Percentage of Superintendents Who Favor the Plan of Giving More Attention to the Topics Listed, Distributed so as to Reveal Differences in Attitude in Large Cities, Cities and Counties, and for all Cities

Topic	Cities of 100,000 and Over	Cities and Counties	All Cities
Addition	59	75	75
Subtraction	50	68	69
Multiplication	59	72	72
Division	56	69	70
Fractions	56	66	65
Percentage	31	51	50
Interest	25	41	39
Saving and loaning money	50	61	61
Banking	38	40	39
Borrowing	22	37	37
Building and loan association	13	46	48
Investments	16	44	44
Bonds and stocks	9	20	20
Taxes	25	53	53
Levies	6	36	35
Public expenditure	28	54	55
Insurance	31	54	55
Profits	28	47	46
Public utilities	34	57	57

Chart IV presents these data in graphic form.

In view of the theoretical advantage of increasing the emphasis upon these subjects and because it corresponds with the experience of the superintendents of this country, it is recommended that additional emphasis be placed upon the following subject matter in arithmetic courses of study: addition, multiplication, subtraction, division, fractions, saving money, public utilities, public expenditures, insurance, taxes, percentage, profits, building and loans, investments, interest, banking, borrowing, levies, stocks and bonds. Additional subjects might be added to this list, and no doubt will be added from time to time. This does not mean that the school should give necessarily the same amount of time to all these subjects. It is moreover conceded that certain of the subjects might be properly united. For instance, public expenditures, public utilities,

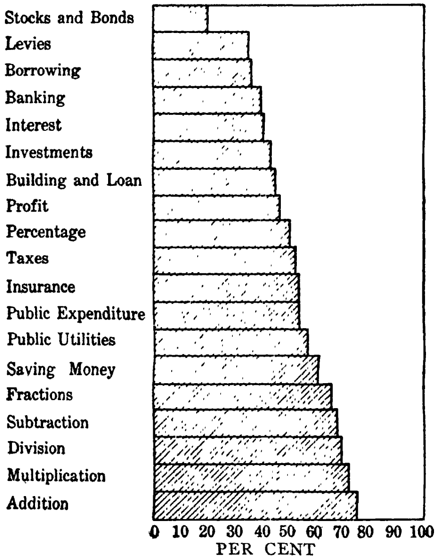

CHART IV.

taxes and levies might be handled all under one head, such as public finances. The important point, however, is that children should be taught to understand the issues involved in the expenditure of public money, the purchase of public utilities, and the levying of taxes. Since the school is to train public officials and the tax payers of the future, it is important that an increased amount of attention be given to such topics as these. Saving money might be handled in connection with banking, building and loans, insurance, or in connection with problems of economical purchasing or expenditures. The point of vital importance is that children understand the conditions involved in these various activities.

It should be noted that the conditions under which children are taught addition, multiplication, subtraction, and division are different from those under which they are taught the arithmetical applications. The method in the one is that of intelligent drill; the method in the other is that of intelligent appreciation, detailed description, definite clearing up of points.

Van Houten's Investigation

As a check upon the returns secured by the questionnaire and also for the purpose of presenting a more detailed analysis of the topics whose retention is regarded as questionable, we include the results of an investigation conducted by Mr. L. H. Van Houten.

TABLE III

Elimination Specifically Mentioned in 148 Courses of Study

Subject	Cases
Annuities	1
Average of accounts	2
Average of payments	2
Denominate numbers:	
Compound numbers	1
Apothecaries' weight	10
Denominate fractions	1
Paper measure	2
Surveyors' measure	10
Troy weight	10
Terms:	
Bale	1
Bundle	1
Chain	1
Dram	1
Folio	1
Furlong	2
Hand	1
Link	1
Quarter, avoirdupois	1
Quintal	1
Rood	2
Sign	1
Decimals, circulating	2
Duties and customs	3
Equation of payments	2
Exchange:	
Bills of	2
Foreign	7
Domestic	5
Foreign money	6
Fractions:	
Comparison	1
Complex except in simple form	4
Compound	1
Government land	1

TABLE III — *Continued*

Subject	Cases
Interest:	
Annual	8
All but 6 % method	7
Compound	10
Exact	2
Interest commission	1
Present worth	3
Bank discount	3
True discount	4
Partial payments	12
Except U. S. rule	3
Except two indorsements	3
Insurance:	
Life	2
Fire	1
Involution and evolution	2
Cube root	44
L. C. M. and G. C. D. except by factor	8
Longitude and time	6
Standard time	1
Measurements:	
Convex surface of and volume of, cone, sphere, pyramid.	3
Circular measure	1
Painting	1
Papering	1
Plastering	2
Roofing	1
Sphere	2
Rhomboid and rhombus	1
Trapezoid and trapezium	2
Round log	1
Surface measures	2
Markings	1
Metric system	14
Notation, scales of	1
Partnership	4
With time	2
Progressions	3

TABLE III — *Continued*

Subject	Cases
Proportion:	
Compound	13
Partitive	3
Percentage:	
Commission and brokerage	1
Finding B when R and P are given	1
Trade discount	3
Stocks and bonds	5
Taxes	2
Poll	1
Temperature	1
Specific gravity	2
Reinvestment and net proceeds	1
Casting out nines	1
Savings bank accounts	1
Optional subjects:	
Investments	1
Ratio	1
Square measure	1
Tariff	1

NOTE. — Mr. L. H. Van Houten, Superintendent of Schools, Toledo, Iowa, made an intensive study of one hundred forty-eight courses of study in arithmetic and then checked his inferences by a questionnaire investigation. His results were accepted as a Master's thesis at the University of Iowa, and are on file in the library of that institution. Mr. Van Houten has granted the authors the privilege of drawing heavily upon his material.

A comparison of the results secured by the two investigations warrants the conclusion that the course of study makers are slightly more progressive when they consider elimination impersonally than when they are publishing courses of study and reports for their con-

stituencies. While this comparison of topics has no particular scientific merit, it does reveal a situation full of human interest. It also confirms the impression secured from the former study that a simplification of texts and courses of study in arithmetic is demanded.

Summary

Summarizing, it should be said that the present tendency among superintendents is in favor of the elimination of questionable matter in the field of arithmetic. While as yet there is not perfect agreement as to just what all of this questionable subject matter is, there is rather clear agreement in regard to certain subjects, such as apothecaries' weight, troy weight, furlong, rood in square measure, dram, quarter in avoirdupois, surveyors' tables, foreign money, folding paper, reduction of more than two steps, long measure, greatest common divisor, least common multiple, true discount, cube root, partnership, compound proportion, compound and complex fractions, annual interest, longitude and time, unreal fractions, alligation, metric system, progression, aliquot parts. Again, the attitude of the superintendents indicates a tendency to give increased attention to the following topics: addition, subtraction, multiplication, division, fractions, percentage, interest, saving and loaning money, banking, borrowing, building and loan associations, investments, bonds and stocks, levying of taxes, public expenditures, insurance, profit, public utilities.

CHAPTER II

THE GRADE DISTRIBUTION OF THE DIFFERENT ARITHMETICAL TOPICS

Factors Influencing Choice and Sequence of Topics

A student of the courses of study in arithmetic is immediately impressed with the fact that wide variation exists in grade distribution of the topics. The choice and sequence of topics may be determined by the mental maturity of the child. Or the choice and sequence may be determined by the logic of the case; that is to say, there may be certain topics in arithmetic which should precede others, because they are preparatory and basic to these later topics. A third factor in the determination of the sequence of the topics in arithmetic may be that of the social and economic conditions affecting the school. For example, while it may be desirable, so far as the maturity of the child is concerned, for us to postpone certain business applications of arithmetic until the later years of the child's school life, we are compelled, nevertheless, to take into consideration the fact that large numbers of children leave school without completing the elementary course;

and consequently it becomes necessary for us to choose between having the child study certain business applications at an early date, or not get them at all.

The supervisor is interested in knowing the adjustment the American schoolmaster has made to these different factors. To what extent has he recognized the limitations of immaturity? To what extent has he accepted the logical demands of the subject matter? To what extent has he recognized the necessity for an early introduction of the social and business applications of arithmetic?

INVESTIGATIONS

Three important investigations have been made in this field with a view toward finding out the adjustment which the school has made in actual practice. Dr. Bruce R. Payne in his study of the elementary school curriculum in 1906 reported on the grade distribution of twenty-nine topics in ten American cities. The Baltimore School Commission in 1911 compared the policy of thirteen American cities with reference to seven selected phases of arithmetic work. Mr. L. H. Van Houten's investigation covers one hundred forty-eight courses of study distributed throughout the United States.

The following table based on the study of Dr. Payne shows the grade distribution in ten American cities.[1]

[1] Payne, *Elementary School Curriculum*, Silver, Burdett & Co.

TABLE IV

TABLE SHOWING TWENTY-NINE TOPICS IN ARITHMETIC AND THEIR DISTRIBUTION BY GRADES IN THE PUBLIC ELEMENTARY SCHOOLS OF TEN AMERICAN CITIES (Payne)

GRADE	I	II	III	IV	V	VI	VII	VIII
Numeration	10	10	10	8	5	3		1
Notation	10	10	10	8	5	3		1
Relation of numbers	7	4	3	1	1			
Addition	8	9	10	8	3	1		
Subtraction	5	9	10	8	3	1		
Multiplication	2	7	7	8	4	1		
Division	2	5	6	8	6	3		
Fractions	3	4	6	8	10	9	3	4
Denominate numbers	6	5	4	9	7	10	6	6
Involution and evolution				1	3	2	2	9
Decimal fractions					4	8	7	3
Mensuration			1		2	2	3	7
Multiplication tables		4	5	4	1	1		
Commission and brokerage							5	
Insurance							5	1
Percentage				1	2	5	7	5
Ratio and proportion						1	3	5
Partnership							2	4
Partial payments							2	4
G. C. M. and L. C. M.				2	5			
Longitude and time							2	2
Profit and loss							4	1
Taxes							5	
Duties							1	
Banking							7	1
Exchange							2	2
Simple interest				1		3	7	5
Stocks and bonds							3	1
Business forms					1	3	1	1

It will be observed that wide overlapping existed. For instance, numeration and notation were taught in seven grades. Fractions and denominate numbers were taught in every grade. Addition, multiplication, division,

and subtraction were taught in six grades. All ten of the cities agree that numeration and notation should be taught in the first three grades; addition and subtraction in the third grade; fractions in the fifth grade; and denominate numbers in the sixth grade. The figures reveal a great disparity of opinion in regard to the other topics.

The following table shows the information gathered by the Baltimore Report.[1]

TABLE V

THE YEAR OF THE COURSE IN WHICH SPECIFIED TOPICS IN ARITHMETIC ARE TREATED IN THE CERTAIN CITIES (Baltimore Commission)

CITIES	FORTY-FIVE COMBINATIONS LEARNED	MULTIPLICATION TABLES LEARNED	LONG DIVISION TAUGHT	ADDITION AND SUBTRACTION OF FRACTIONS TAUGHT	MULTIPLICATION AND DIVISION OF FRACTIONS TAUGHT	DECIMALS TAUGHT	PERCENTAGE TAUGHT
New York	2	3	3	4	5	5	6
Chicago	2	4	4	5	5	6	6
Philadelphia	2	2	3	5	5	6	6
St. Louis	2	3	4	3	4	4	5
Boston	2	4	4	5	6	5	6
Cleveland	2	4	4	5	6	5	6
Baltimore	2	3	4	3	4	4	6
Pittsburgh	2	3	4	4	5	5	6
Detroit	2	3	4	4	5	5	6
Buffalo	2	3	4	5	5	5	6
San Francisco	2	3	4	4	5	4	6
Milwaukee	4	4	4	5	5	6	7
Cincinnati	2	3	3	4	5	4	6

[1] Report of the Commission Appointed to Study the System of Education in the Public Schools of Baltimore. Bureau of Education Bulletin, 1911, No. 4, p. 76.

The Commission drew the following conclusions from the foregoing table: "From the above table it appears that the forty-five combinations in arithmetic are learned in all but two of the cities in the second grade. The most common grade in which the multiplication tables are learned is the third. Long division is taught most commonly in the fourth grade; addition and subtraction, multiplication and division of fractions, and decimals are taught most commonly in the fifth grade; and percentage in the sixth."

Addition and subtraction of fractions, however, are taught in these cities in grades as low as the third and as high as the fifth; multiplication and division of fractions appear in grades as low as the fourth and as high as the sixth; decimals are taught in grades as low as the fourth and as high as the sixth.

The investigation of Dr. Payne and the Baltimore Commission related to the grade distribution of topics in a small group of large cities only. The most thoroughgoing investigation of the problem was made by Mr. L. H. Van Houten in 1913.

The Van Houten Investigation

Mr. Van Houten based his report upon one hundred and forty-eight courses. Cities ranging in population from 1118 to that of New York City are included in his report. These cities are located in thirty-seven

states and in the District of Columbia. State courses of study were obtained from nearly every state and were used when referred to by the city courses.

Some of the topics selected were chosen on account of their commonly accepted importance and others because of the discussion which has arisen concerning their elimination. Care was taken to verify all the material so far as possible by consulting the texts in use when the course was not sufficiently specific. In many cases the text was the sole guide, since only text assignments were given.

TABLE VI

FREQUENCY TABLE SHOWING GRADE OCCURRENCE OF ARITHMETIC TOPICS IN 148 CITIES (Van Houten)

Topic	Grade 1	2	3	4	5	6	7	8	9	Modal Grade
Notation	98	126	139	129	96	58	37	17		3
Numeration	101	125	139	132	97	58	38	18		3
Addition	108	148	148	145	117	90	73	39		3, 4
Subtraction	98	143	146	145	125	88	74	10		3
Multiplication	38	108	147	145	117	90	73	41		3
Division	28	97	146	145	117	91	68	41		3
Factoring			3	30	65	35	9	2		5
Cancellation			17	62	43	3	2			4
Fractions	69	110	124	138	147	104	82	43		5
Denominate numbers	67	94	102	97	103	121	72	34		6
Involution and evolution					2	2	13	123	2	8
Decimal fractions				42	121	122	63	27		5
Mensuration	83	85	90	93	92	121	95	96		6
Multiplication tables	9	89	132	92	13	4	4	3		3

TABLE VI — *Continued*

Topic	Grade 1	2	3	4	5	6	7	8	9	Modal Grade
Commission and brokerage					1	43	86	36		7
Insurance					1	6	84	52		7
Percentage			1	12	42	105	108	62		7
Ratio and proportion				2	7	21	53	91	3	8
Partnership							10	44	3	8
Partial payments						1	33	60	3	8
G. C. D. and L. C. M.						1	1	6		8
Longitude and time					1	15	34	51	1	8
Profit and loss					1	49	90	20		7
Taxes					1	7	83	55	1	7
Duties						1	67	52	1	7
Banking							23	58	2	8
Exchange							28	72	2	8
Simple interest				6	9	72	117	61	1	7
Stocks and bonds						2	28	73	2	8
Business forms			2	39	62	66	44	30		6
Simple accounts			1	15	31	39	37	30	1	6
U. S. money	48	54	99	85	71	45	16	5		3
Approximations					4	11		2		6
Commercial discounts				7	10	46	82	46	1	7
Bank discount					1	12	51	67	3	8
Analysis					9	15	59	58		7
Metric system				1	2	2	22	80		8

NOTE. — This table should be read as follows: Notation is taught in the first grade in 98 cities, in the second grade in 126 cities, etc.

From the foregoing it may be seen that wide variation exists in practice — in fact addition is the only topic reported in all of the cities in either the second or third grade. Subtraction, multiplication, and division are almost uniformly taught in the third and fourth grades. Only eleven topics are taught in the first and

second grades in any of the cities, while almost one third of the cities did not teach any of the topics in these grades.

Although five additional topics are taught in a few schools in the third grade, the greatest agreement is on teaching notation, numeration, addition, subtraction, multiplication, division, fractions, denominate numbers, multiplication tables, and United States money.

In the fourth and fifth grades there is a decided increase in the number of cities teaching factoring, cancellation, decimal fractions, simple accounts, and business forms.

The upper grades show a marked tendency to select fewer of the formal topics and more of the topics which have to do with the application to social and economic situations.

Perhaps the most striking fact of the table is that there seems to be so little agreement in grade distribution of topics. Such variation suggests the desirability of our knowing more about the relative success of the different schemes of distribution.

Summary

The following is a summary showing the modal grade in which the various topics are taught:

Grade I. Variation is such that no mode appears.
Grade II. Variation is such that no mode appears.

Grade III.	Notation Numeration Addition Subtraction Multiplication Division Multiplication tables U. S. money
Grade IV.	Addition Cancellation
Grade V.	Factoring Fractions Decimal fractions
Grade VI.	Denominate numbers Mensuration Business forms Simple accounts
Grade VII.	Commission and brokerage Insurance Percentage Profit and loss Taxes Duties Simple Interest Commercial discounts Analysis

Grade VIII.	Involution and evolution Ratio and proportion Partnership Partial payments G. C. D and L. C. M. Longitude and time Banking Exchange Stocks and bonds Bank discounts Metric system

It is thus seen that the four fundamental processes are most frequently taught in the first four grades.

Factoring is taught most frequently in the fifth grade.

Cancellation is taught most frequently in the fourth grade.

Fractions are taught most generally in the fifth grade.

Decimal fractions are taught most frequently in the fifth and sixth grades.

Denominate numbers and mensuration are taught quite generally throughout the whole course of eight grades, although there is a tendency for this material to be taught in the sixth grade or lower.

Involution and evolution are taught predominately in the eighth grade.

Percentage and its applications appear most frequently in the sixth and seventh grades.

The metric system is taught most generally in the eighth grade.

Grade Occurrence of Seven Specified Topics

Table VII was made by taking the seven specified topics used by the Baltimore Commission's report. It supplements the information given in the preceding table, by telling definitely where emphasis is placed on a subject. In this table no grades are mentioned unless the topics receive their principal treatment there. In cities where two grades seemed to be of equal importance, both are given.

As found by the Baltimore Commission in a study of the courses of ten cities, the forty-five combinations are completed by a majority of schools in the second grade. About ten per cent complete these in the third grade and only four per cent in the first grade. The multiplication tables are generally completed in the third grade, though there is greater variation than in the combinations. The fourth grade is the grade of long division. About twenty per cent of the schools teach the topic in the third grade, and three per cent in the fifth. Formal fractions are taught in the fifth grade by the majority of schools. Twenty-eight schools teach addition and subtraction of fractions and multiplication and

division in different grades. In nineteen of these cases the first two processes are taught in the fourth grade and the latter in the fifth. Decimal fractions likewise have the fifth as the modal grade, though over forty per cent of the schools offer the subject in the sixth grade. In seventeen cities they are continued through two grades. Grade six is the time when the principles of percentage are studied.

The following table shows the practice in the cities studied:

TABLE VII

THE YEAR OF THE COURSE IN WHICH SEVEN SPECIFIED TOPICS IN ARITHMETIC ARE TREATED (Van Houten)

Cities	Topics						
	1	2	3	4	5	6	7
Aberdeen, S. Dak.	2	3	4	5	5	5	6
Akron, Ohio	2	3	4	5	5	6	6
Albia, Iowa	2	4	4	5	5	6	6
Altoona, Pa.	2	4	4	6	6	6	6
Ansonia, Conn.	2	3	3	5	5	5	5
Appleton, Wis.	2	4	4	5	5	5	6
Astoria, Ore.	2	3	3	4	5	5	6
Athens, Ga.	2	2	4	5	5	5	6
Atlanta, Ga.	2	3	4	5	5	5	7
Atlantic, Iowa	2	4	4	5	5	7	8
Baraboo, Wis.	2	4	4	5	5	5 & 6	6
Belleville, Ill.	2	3	4	5	5	5	6
Berkeley, Cal.	3	4	4	5	5	5	6
Birmingham, Ala.	2	3	4	4	5	5	6
Boise, Idaho	2	3	4	5	5	6	6
Boone, Iowa	2	3	4	4	5	5	6
Boston, Mass.	?	4	4	5	5	5	6
Boulder, Colo.	2	3	3	4	5	5	7

TABLE VII — *Continued*

Cities	Topics						
	1	2	3	4	5	6	7
Bowling Green, Ky.	2	3	4	4	5 & 6	6	7
Bradford, Pa.	2	3	4	5	5	5	7
Brockton, Mass.	?	3	4	5	6	6	7
Burlington, Iowa	2	3	4	5	5	5	7
Cambridge, Mass.	2	3	4	4	4	5	6
Canton, Ohio	2	4	5	5	6	6	7
Centerville, Iowa	2	3	3	5	5	5	7
Chester, Pa.	2	4	4	4	5	5	6
Chicago, Ill.	2	3	5	5	5	6	7
Cheyenne, Wyo.	2	3	4	5	5	5	6
Cincinnati, Ohio	2	3	4	5	6	6	7
Cleveland, Ohio	2	4	4	5	5	6	7
Columbus, Ga.	2	3	4	5	5	5	6
Connersville, Ind.	2	4	4	5	5	5	2
Cortland, N. Y.	2	3	4	5	5	5	6
Covington, Ky.	2	3	4	5	5	7	7
Crawfordsville, Ind.	2	4	4	5	5	5	6
Danbury, Conn.	2	3	4	5	5	5	6
Davenport, Iowa	2	3	3	5	5	5	6
Detroit, Mich.	2	3	4	5	5	5	6
Dover, N. H.	2	4	4	5	5	5	6
Dubuque, Iowa	1	3	4	4	5	5	6
Dunkirk, N. Y.	1	3	3	4	5	5	6
Eau Claire, Wis.	2	4	4	5	5	5	6
Elgin, Ill.	2	4	4	5	5	5	7
Englewood, N. J.	2	3	3	4	4	5	6
Enid, Okla.	2	3	4	5	5	5	6
Erie, Pa.	2	3	4	4	4	5	7
Everett, Wash.	2	4	4	5	5	6	6
Fort Smith, Ark.	2	3	4	5	5	5	6
Fort Wayne, Ind.	3	4	4	5	5	6	6
Frankfort, Ky.	1	2	3	4	5	5	7
Freeport, Ill.	3	3	4	5	5	5 & 6	6
Fresno, Cal.	3	4	5	5	5	5 & 6	7
Fulton, N. Y.	2	3	4	5	5	5	6
Galesburg, Ill.	2	3	4	5	5	5	5

TABLE VII — *Continued*

Cities	Topics						
	1	2	3	4	5	6	7
Gloversville, N. Y.	2	3	4	6	6	6	7
Grand Junction, Colo.	2	4	4	5	5	5	6
Guthrie, Okla.	2	4	4	5	5	5	6
Harrisburg, Pa.	3	3	4	4	4, 5 & 6	5	8
Hartford, Conn	2	3	4	5	5	6	6
Indianapolis, Ind.	3	4	4	5	5	6	6
Ironwood, Mich.	3	4	4	5	5	6	5 & 6
Jamestown, N. Y.	2	3	4	5	6	6	7
Jefferson, Iowa	3	4	4	5	5	6	6
Jersey City, N. J.	2	3	3	5	5, 6 & 7	5	6
Johnstown, Pa.	3	4	4	5	5	5	6
Lancaster, Ohio	2	4	4	5	5	6	6
Lansing, Mich.	2	3	4	4	4, 5 & 6	4 & 5	4 & 5
Laramie, Wyo.	2	4	4	5	5	5	6
Lincoln, Neb.	2	4	4	5	5	6	6 & 7
Long Beach, Cal.	2	4	4	5	6	5	7
Los Angeles, Cal.	3	4	4	5	6	5	7
Lynn, Mass.	2	3	5	5	5	6	6
Madison, Wis.	2	4	4	5	5	6	6
Manchester, N. H.	2	3	4	4	5	6	6
Manistee, Mich.	2	4	4	5	5	6	7
Marengo, Iowa	2	3	4	5	5	5	6
Marion, Ind.	2	4	4	6	6	6	7
Mason City, Iowa	2	3	4	5	5	5	6
Memphis, Tenn.	2	3	4	4	5	6	
Menominee, Mich.	2	3	3	4	4	4 & 6	6
Milwaukee, Wis.	2	3	4	5	5	6	7
Minneapolis, Minn.	2	4	4	5	5	6	6 & 7
Monessen, Mass.	2	3	4	5	5	5	6
Muscatine, Iowa	2	4	4	5	5	6	6
Muncie, Ind.	2	4	4	5	5	5	7
Muskogee, Okla.	2	4	4	6	5 & 6	6	6 & 7
Norfolk, Va.	2	3	4	6	6	6	7
Nashville, Tenn.	2	3	4	4	4	5	5
New Hampton, Iowa	2	4	3	5	5	5	6
New Haven, Conn.	2	4	4	5	5	5	6

TABLE VII — *Continued*

Cities	Topics						
	1	2	3	4	5	6	7
Newton, Mass.	2	3	4	4 & 5	6	5, 6 & 7	7
New York, N. Y.	2	4	4	5	5	5	6
Niles, Ohio	2	4	4	5	5	6	6
Oakland, Cal.	3		4	5	5	5	6
Oklahoma City, Okla.	2	4	4	5	5	5	6
Olean, N. Y.	2	3	3	5	5	5	6
Ownesboro, Ky.	2		4	5	5	5	5
Paterson, N. J.	2	3	4	5	5	6	7
Pensacola, Fla.	2	4	4	5	5	5	7
Philadelphia, Pa.	2	3	3	5	5	6	6
Phoenix, Ariz.	2	4	5	6	6	6	7
Piqua, Ohio	2	3	3	4	5	5	6
Plainfield, N. J.	2	3	4	4	5	5	5
Pomona, Cal.	3	3	4	5	5	5	6 & 7
Portland, Me.	2	3	4	4	5	5	6
Providence, R. I.	2	3	4	5	5	6	7
Raleigh, N. C.	2	3	3	4	4	5	6
Reading, Pa.	2	3	3	5	5	6	6
Reno, Nev.	2	3	3	4	5	6	6
Richmond, Va.	2	3	4	5	5	6	6
Riverside, Cal.	3	4	4	5	5	6	7
Roanoke, Va.	2	3	3	4	4	3 & 4	6
Saginaw, Mich.	2	3	4	5	5	5 & 6	7
Salem, Ore.	2	4	4	5	6	5 & 6	7
Salt Lake City, Utah	2	3	4	5	5	6	6
San Francisco, Cal.	2	3	3	4 & 5	5	6	6
San José, Cal.	2	3	4	5	5	6	7
Sault Ste. Marie, Mich.	2	4	4	5	5	5	6
Schenectady, N. Y.	2	3	4	5	5	5	6
Scranton, Pa.	2	3	4	5	6	6	7
Sedalia, Mo.	2	3	4	5	5	5	6 & 7
Shamokin, Pa.	2	2	3	4	4	5	6
Sheboygan, Wis.	2	3	3	4 & 5	5	5	6
Spokane, Wash.	2	3	4	4	5	5	6
Springfield, Mass.	2	4	4	4	5	5	5 & 6
Springfield, Mo.	1	2 & 3	3	4	4	5	6

TABLE VII — *Continued*

Cities	Topics						
	1	2	3	4	5	6	7
Springfield, Ohio	2	3	4	5	5	5 & 6	5 & 6
St. Joseph, Mo.	3	4	4	5	5	6	6
St. Paul, Minn.	2	4	4	5	5	6	6
Superior, Wis.	2	4	4	5	5	5 & 6	6
Syracuse, N. Y.	2	3	4	4	5	5	6
Tacoma, Wash.	2	3	4	5	5	6	6
Terra Haute, Ind.	2	4	4	5	5	6	7
Tulsa, Okla.	2	4	4	5	5	5	6
Vincennes, Ind.	2	4	4	5	5	5 & 6	6
Washington, D. C.	2	3	4	5	5	6	7
Watertown, N. Y.	2	3	3	4	5	5	6
Wausau, Wis.	2	3	3	4	5	6	7
Webb City, Mo.	3	3	4	5	5	5 & 6	6
West Chester, Pa.	2	3	4	5	5	5	6
Winston, Pa.	?	3	3	5	5	6	7
Worcester, Mass.	2	3	4	5	5	5	6
Utica, N. Y.	1	3	4	5	5	5	6
York, Pa.	2	3	4	5	5	6	6

The foregoing table should be read thus: In Aberdeen, South Dakota, the forty-five combinations are completed in the second grade; the multiplication tables are learned in the third grade. Long division is taught in the fourth grade. Addition and subtraction of fractions are taught in the fifth grade. Multiplication and division of fractions are taught in the fifth grade. Decimals are taught in the fifth grade; percentage is taught in the sixth grade. Column 1 represents the forty-five combinations learned; 2, Multiplication tables learned; 3, Long division taught; 4, Addition and subtraction of fractions taught; 5, Multiplication and division of fractions taught; 6, Decimals taught; 7, Percentage taught.

The table below is a summary of the foregoing and shows the prevailing practice in these particulars.

TABLE VIII

FREQUENCY TABLE SHOWING GRADE OCCURRENCE OF SEVEN SPECIFIED TOPICS

Cities	Grade 1	2	3	4	5	6	7	8
Forty-five combinations completed	5	122	14	8				
Multiplication tables completed		3	87	53				
Long division taught			26	107	5			
Addition and subtraction of fractions taught				29	108	13		
Multiplication and division of fractions					120	18	8	
Decimals taught					91	66	3	
Percentage taught					9	93	46	2

SUMMARY

In summarizing this, Mr. Van Houten says:

"The forty-five combinations are completed by a majority of schools in the second grade. About ten per cent complete these in the third grade and only four per cent in the first grade. The multiplication tables are generally completed in the third grade, though there is greater variation in this particular than in the completion of the forty-five combinations. The fourth

grade is the grade of long division. About twenty per cent of the schools teach the topic in the third grade, and three per cent in the fifth. Formal fractions are taught in the fifth grade by the majority of schools. Twenty-eight schools teach addition and subtraction of fractions and multiplication and division of fractions in different grades. In nineteen of these cases the first two processes are taught in the fourth grade and the latter in the fifth. Decimal fractions likewise are taught in grades five, six, and seven, yet the fifth grade is predominantly the grade in which decimal fractions are taught. Although percentage appears in the sixth, seventh, and eighth grades, and in some cases as low as the fifth, yet the sixth grade is the most frequent grade in which percentage is taught."

CHAPTER III

TIME ALLOTMENT FOR ARITHMETIC

Varying the Emphasis upon Arithmetic

Arithmetic has been held in varying degrees of esteem in the past. At certain stages of civilization little attention was given to the quantitative interpretation of the environment of the individual. The early Massachusetts enactment of 1647 in regard to education ordered that reading and writing schools be established in every community of fifty householders. No doubt a certain amount of arithmetic was taught in these early reading and writing schools, yet the facts indicate that arithmetic when taught at all was taught incidentally. No formal recognition of the subject was given in connection with the time allotment in the daily program.

Even though arithmetic was included in the "Seven Liberal Arts," it should be borne in mind that, as a subject for elementary education for the masses, it was not considered of importance until relatively late. The demand for a knowledge of arithmetic came with the development of commerce in New England in the latter

part of the eighteenth century and with the expansion of commercial activity in the nineteenth century.

The early grammar school, both in New England and elsewhere in this country, gave arithmetic a place. The arithmetic of the grammar school period was of an elementary character, and was not infrequently differentiated into several parts. This tendency to differentiate the subject matter of arithmetic into various parts was even more strikingly emphasized during the "academy period" (1780–1835), when courses were offered in navigation, accounts, bookkeeping, and the like. The academy gave the subject a definite allotment of time.

With the establishment of high schools in Boston, New York, and elsewhere, arithmetic received definite recognition. It was included in the list of subjects for which provision was made by the Massachusetts law of 1827 for all communities of fifty or more families or households, and from this time on it was almost invariably included in the curriculum. Not only was arithmetic taught in what we would call to-day the elementary schools, but it was included in the curriculum of the Boston Latin School, the Boston English High School, the Boston Girls' High School, and the Leicester Academy.

Arithmetic was required in all elementary schools in the state of Massachusetts by the law of 1789. The

fact is that with the rise of the academy and the high school it received definite recognition in the daily program of almost every school. With the establishment of the "Common Schools" throughout the United States, which accompanied the development of the West, arithmetic was included as an essential part of every public school curriculum.

While legislation and public opinion both responded to the social demands that arithmetic should be taught, it remained for the schoolmaster to determine the amount of time it should receive. As the general public from time to time placed new emphasis upon the arithmetical attainments of the pupils, the schoolmaster tended to devote more and more time to the subject, with the belief that increased proficiency depended upon an increased amount of time being allotted to the subject. Thus the popular criticism to the effect that the students were inefficient in arithmetic resulted in many cases in an increase in the number of school years in which arithmetic was studied. This adjustment has been questioned recently.

A few years ago Mr. J. M. Rice undertook to analyze the conditions under which arithmetic was being taught. He found that wide variation existed in the amount of time which cities were devoting to the subject. After analyzing the results of a somewhat elaborate series of tests which he gave to elementary school children in a

number of cities, he arrived at the following conclusions: "The amount of time devoted to arithmetic in the school that attained the lowest average, twenty-five per cent, was practically the same as in the one where the highest average, eighty per cent, was obtained. In the former the regular time for arithmetic in all grades was forty-five minutes a day, but some additional time was given. In the latter the time varied in the different grades, but averaged fifty-three minutes daily. The schools showing the most favorable results cannot be accused of making a fetish of arithmetic. These statements are further justified by the fact that the four schools which on the whole stood highest gave practically the same amount of time to arithmetic as the schools which stood lowest."[1]

Dr. C. W. Stone[2] later conducted an investigation on "Arithmetical abilities and some of the factors determining them," partly for the purpose of determining whether the amount of time given to the arithmetic recitation was a factor in arithmetical efficiency. He found among the thirteen schools that devoted less than the median time to the arithmetic recitation that the work was slightly better than in the thirteen schools which devoted more than the median time. Dr. Stone says: "What is claimed is that as the present practice

[1] Rice, *Scientific Management in Education*, p. 119.

[2] Stone, *Arithmetical Abilities*, p. 62.

goes, a large amount of time spent on arithmetic is no guarantee of a high degree of efficiency in arithmetic. If one were to choose at random among the schools with more than the median time given to arithmetic, the chances are about equal that he would get a school with inferior products: and conversely, if one were to choose among the schools with less than the median time given, the chances are about equal that he would get a school with a superior product in arithmetic."

While the foregoing statements do not dispose of the debatable points as to the amount of time which may be given profitably to arithmetic in the public schools, they do tend to shake our faith in the belief that increased arithmetical efficiency is a function of time expenditure. Experimental education will no doubt have additional contributions to make to this problem within a comparatively short time. In the meantime, however, the school superintendent who wishes to profit by the experience of other superintendents is interested in the policy of schools throughout the country with reference to this matter.

One of the important contributions to this problem has been made by Dr. Bruce R. Payne. Dr. Payne undertook to discover the relative amount of time which was being given to arithmetic recitations in the leading cities in this country in 1904. He collected and collated data with regard to the percentage of recitation

time given to each of the subjects in the elementary school curriculum in the following cities: New York, Boston, Chicago, Cleveland, San Francisco, Columbus, Ga., Louisville, Jersey City, New Orleans, and Kansas City, Kan. The following table shows the average per cent of recitation time given to arithmetic in each grade in these cities:

TABLE IX

	I	II	III	IV	V	VI	VII	VIII	Average
Arithmetic .	13 6	15 4	18 2	18 0	17 6	18 3	17.7	17.0	16.975

He found that arithmetic was second only to reading and literature in the amount of time expenditure in the daily programs of these cities.

In the investigation of this same topic in 1911 by the committee appointed to study the system of education in the public schools of Baltimore, of which Dr. E. E. Brown, then commissioner of education of the United States, was chairman, it was discovered that there was a wide variation in the total amount of time given to the study of arithmetic and algebra in New York, Chicago, Philadelphia, St. Louis, Boston, Cleveland, Baltimore, Pittsburgh, Detroit, San Francisco, Milwaukee, and Cincinnati. The range of variation in recitation time per week was from 1125 minutes in St.

Louis to 2080 minutes in Cincinnati. The percentage of school time devoted exclusively to the study of arithmetic and algebra in these cities varied from 10 per cent in Chicago to almost 19 per cent in Cincinnati. This variation, in view of the findings of Mr. Rice and Mr. Stone, suggests the probability that the time given to arithmetic is not being used to an equal advantage in all these cities. It is possible that Chicago, which is giving only 10 per cent of its time to arithmetic, is getting just as good results as Cincinnati, which is giving almost twice as much time to arithmetic.

The material which is presented in the following tables indicates that these cities have been doing more or less experimenting with this important problem.

TABLE X

PER CENT OF RECITATION TIME GIVEN TO ARITHMETIC
(Payne)

	1888	1904		1888	1904
New York . . .	26.2	12.0	St. Louis	19.3	15.2
Boston	16.6	16.2	Louisville . . .	16.7	17.2
Chicago	9.3	18.6			

It should be noted in the above that New York and St. Louis in the period from 1888 to 1904 made definite attempts to decrease the relative time given to arithmetic recitations, while Chicago increased the percent-

age of time. Boston and Louisville remained practically the same.

The Baltimore Commission furnished the following table, which shows conditions in 1890 and 1910 and 1911:

TABLE XI

PER CENT OF RECITATION TIME GIVEN TO ARITHMETIC (Baltimore Survey)

	1890	1910–11		1890	1910–11
New York	26.2	13.4	Detroit	17.2	16.0
Boston	16.6	15.5	San Francisco	14.0	16.6
Chicago	9.3	10.0	Milwaukee	15.5	14.7
St. Louis	19.3	15.0	Cincinnati	13.4	18.8
Cleveland	14.1	15.5	Average	16.51	15.38
Baltimore	19.5	18.3			

It is interesting to note in the above that more than one half of the cities reduced the per cent of time expenditure during the twenty years included in this comparison. New York, which in 1890 was giving 26.2 per cent of her total time to arithmetic, dropped back to 13.4 per cent in 1910. The average time given by the ten cities in 1890 was 16.51 per cent, and in 1910–11 it was 15.38 per cent.

The situation in foreign countries does not differ greatly from the situation in America. Dr. Payne collected data from ten leading cities in England, ten in Germany, and ten in France, showing an average per cent of time given to the recitation of arithmetic.

TABLE XII

	Grades								
	I	II	III	IV	V	VI	VII	VIII	Averages
England .	19.8	19.4	19.4	22.7	21.3	20.5	18.9	16.5	19.87
Germany .	21.2	22.3	18.7	17.6	15.6	15.2	15.2	15.3	17.64
France .	—	12.5	12.5	16.6	16.6	16.6	16.8	—	15.26

Dr. Payne found that "in arithmetic the English courses show 3 per cent more time than American, with about 5 per cent more in the earlier grades." He says that as in America the relative time assigned to arithmetic makes it second in importance to language. The variation between ten German cities is about the same as between ten American cities. The relative time ranges from 14 per cent to 22.8 per cent in the German cities, and between 12 per cent and 19.5 per cent in American cities. The above table shows that the French schools devote a smaller proportion of time to arithmetic than do the schools of the other countries.

In 1913 a study was made by the authors of the amount of time devoted to recitations in arithmetic in six hundred and thirty cities in the United States having a population of four thousand and over. It was found that the variation in the number of minutes per week given to arithmetic ranged from no time at all up to 450 minutes. The time distribution for the different grades is given in the table below:

TABLE XIII

No of Minutes per Week	Grades								
	1	2	3	4	5	6	7	8	9
0	136	31	4					24	439
15	7	2							
20									
25	18	1	1	1	1				
30	10	7	5	1		1	1		
35	1								
40	6	4	2	5	2	1		1	2
45									
50	99	36	3	1	4	4	2	2	
55									
60	27	22	10	5	3	4	5	5	4
65	1								
70	3	3	1	1	1	1	2	1	1
75	103	127	59	12	4			2	
80	6	5	10	11	2	3	1	1	1
85									
90	5	6	12	3	4	2	3	3	2
95									
100	108	83	156	129	71	31	14	10	3
05									
10	1	1	6	5	3			2	1
15									
20	6	10	13	13	12	19	22	18	4
125	17	40	90	113	122	109	59	36	5
30		3	4	5	6	9	3		2
35									
40			2	4	6	1	9	8	3
45									
150	53	66	98	129	158	196	210	192	30
55									
60	1	2	4	4	6	2	7	8	5

TABLE XIII — *Continued*

No. Minutes per Week	Grades								
	1	2	3	4	5	6	7	8	9
65									
70			2	3	3	1		2	
175	6	8	9	20	31	28	35	37	6
80	2	5	6	3	5	9	16	15	4
85									
90		1				1			
95									
200	11	33	54	56	52	72	96	121	60
05									
10		3	2		1		2	5	7
15									
20		1	1	2	1	1	1	2	2
225	4	10	25	19	22	28	31	38	27
30				1	1	2	2	2	
35									
40		4	8	8	4	4	4	7	1
45									
250	1	9	25	39	51	51	45	34	13
55									
60		1		1	1	2	2	2	
65									
70			1	1	4	3	2	1	1
275		2	2	8	7	9	8	5	1
To 300		4	14	24	30	30	38	33	5
To 350				3	2	3	7	8	2
To 450	1	1	1				2	3	

Appendix A shows interesting variations in the time schedules of forty selected cities.

From left to right this table means that there are 136 cities that give no time to recitations in arithmetic in the first grade, 31 cities that give no time to it in the second grade, and 4 that ignore it in the third grade. From the top down the table reads: 136 cities give no time to arithmetic recitations in the first grade; 7 give fifteen minutes per week in the first grade; 18, twenty-five minutes; 10, thirty minutes, etc. The range for the first grade is from no time to 450 minutes per week, and it is practically the same for the other grades.

The median number of minutes per week devoted to recitations in arithmetic in the different grades is shown in the following table:

TABLE XIV

Grades	I	II	III	IV	V	VI	VII	VIII
Median number of minutes per week	75	100	125	150	150	150	150	170

Chart V shows graphically the difference in variation and the central tendency in practice.

Attention is directed to the upper and lower quartiles. It will be noted that the cases included within the middle 50 per cent are close to the median. The lower 25 percentile shows that one fourth of the cities spend 25 minutes or less in the first grade; 75 minutes or less in the second grade; 100 minutes or less in the third

and fourth grades; 125 minutes or less in the fifth and sixth grades; and 150 minutes or less in the seventh and eighth grades. Again, the upper quartile brings out the fact that another fourth of the cities spend 100 minutes or more in the first grade; 125 minutes or more

MEDIAN MINUTES PER WEEK GIVEN TO RECITATIONS IN ARITHMETIC

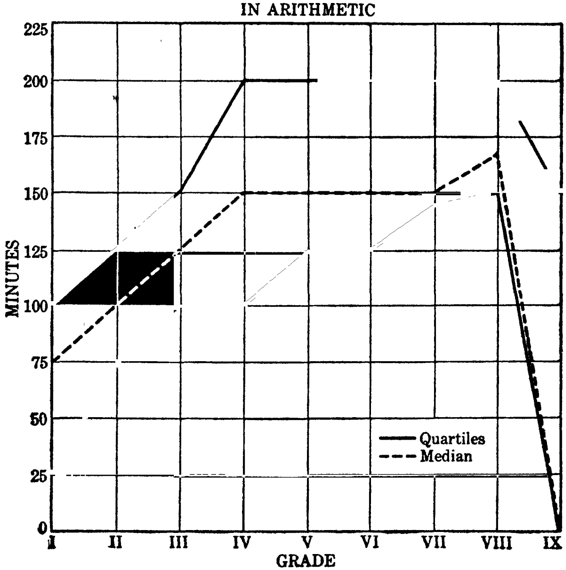

CHART V.

in the second grade; 150 minutes or more in the third grade; 200 minutes or more in the fourth, fifth, sixth, seventh, and eighth grades. From the foregoing analysis it may be seen that some cities spend relatively far more recitation time than others on arithmetic.

Comparison of the 150 cities spending the greatest amount of time with the 150 cities spending the least amount of time indicates that already many of these cities are making headway in the economy of time. If one fourth of the cities can get satisfactory results with an expenditure of from 5 to 20 minutes per day or less in each of the first four grades, there is reason for inquiry as to the accomplishment of cities which spend from 20 to 40 minutes or more per day in these grades. Again, if one fourth of the cities are able to get satisfactory results in from 20 to 30 minutes per day or less in the fifth to eighth grades, certainly we have cause to question the reason why another one fourth of the cities spend from 40 to 60 minutes or more per day in those grades. On the whole, it seems safe to say that the wide variation of recitation time in the various cities of the United States suggests the possibility of attempting to effect an economy of time by means of standardizing the number of minutes in the recitation period. It is true that we need a comparison of the actual results obtained in all cities giving a large amount of time to the recitation of arithmetic, with all cities giving a small amount of time. In the absence of scientific data touching all cities, however, it is important that we take cognizance of the fact that the investigations which have thus far been made by Rice, Stone, and others, all indicate that there

is no assurance of doubling the efficiency in arithmetic by doubling the recitation time. Therefore, it seems safe to recommend that the supervisor modify practice in a particular city on the basis of the experience of supervisors elsewhere, and such experimental evidence as has been obtained.

If those cities spending more than the median amount of recitation time would reduce their schedule to the median time, a decided economy might be effected. In doing this no one would be adopting an untried policy, as the 157 superintendents who are included within the first quarter have already adopted this policy. It is difficult to believe that the results attained in the 315 schools reporting the median time expenditure are distinctly inferior to the results attained in the other schools.

Further analysis of these data indicates that the North Atlantic and South Atlantic cities spend from 25 per cent to 50 per cent more time in arithmetic than do the cities in the other parts of the country. Large cities seem to give more time to arithmetic recitations than do small cities, although the difference is not great. The county superintendents report a very much smaller amount of time given to the arithmetic recitations in the rural schools. This is, no doubt, due to a difference in the organization of the daily program in the rural schools. However, it should be noted that no

section or city differs so much from the central tendency as to make it difficult to work toward some part of the lower half of the curve of time distribution for recitations in each grade. That is to say, each city might wisely try the following time expenditure:

TABLE XV

NUMBER OF MINUTES OR LESS PER WEEK IN EACH GRADE

GRADES	I	II	III	IV	V	VI	VII	VIII
Median minutes .	75	100	125	150	150	150	150	170

This table becomes clear when read as follows: The median time expenditure per week in the first grade is 75 minutes, or 15 minutes per day; in the second grade 100 minutes, or 20 minutes per day; in the third, 125 minutes, or 25 minutes per day; in the fourth, fifth, sixth, and seventh grades 150 minutes, or 30 minutes per day; in the eighth grade 170 minutes per week, or a little less than 35 minutes per day.

RELATIVE TIME EXPENDITURE

The supervisor who adopts the standard proposed above may do so with the assurance which comes with the knowledge that this is the generalized experience of other teachers, principals, and supervisors. This

standard more nearly represents the judgment of the American schoolmaster, based on generations of experience in teaching arithmetic, than does any other standard.

The amount of time arithmetic should receive in relation to the other subjects of study is a factor of importance in making school programs. A critical study of the time distribution in one hundred forty-eight cities was made by Mr. Van Houten for the purpose of determining this relationship. The material was first distributed to show the total number of minutes per week given to arithmetic. This distribution is shown in the next table.

The meaning of this table becomes clear when read thus: Akron, Ohio, spent 250 minutes per week on arithmetic in the first grade, 300 minutes in the second grade, 300 minutes in the third grade, 300 minutes in the fourth grade, 275 minutes in the fifth grade, 275 minutes in the sixth grade, 300 minutes in the seventh grade, 300 minutes in the eighth grade. In all 2300 minutes were given over to arithmetic. 11,100 minutes were given over to recitation in all subjects each week. Arithmetic received 20.7 per cent of the total time.

Table XVI is a summary of Table XV.

TABLE XV

Table Showing Time Spent on Arithmetic in Each of the Grades Below the High School, the Total Time Spent in the Grades, the Total Recitation Time Spent on all Subjects, and the Percentage of the Total Time Spent on Arithmetic

City	State	Grade 1	2	3	4	5	6	7	8	9	Total Minutes per Week Arithmetic	Total Minutes per Week All Subjects	Percentage of Total
Akron	Ohio	250	300	300	300	275	275	300	300		2300	11,100	20.7
Albia	Iowa	150	150	250	250	300	300	275	275		1950	10,920	17.8
Altoona	Pa.	125	150	150	150	200	200	200	150		1325	10,325	10.8
Ann Arbor	Mich.		70	110	130	135	120	150	250		965	10,680	8.0
Ansonia	Conn.	155	300	300	300	300	300	300	300		2255	10,965	20.5
Astoria	Ore.	75	75	100	125	150	150	150	150		975	11,205	8.7
Athens	Ga.	100	100	150	150	150	180	180			1250	8,600	14.5
Atlanta	Ga.	120	175	175	200	300	300	250	300		1820	12,000	16.5
Atlantic	Iowa	50	75	100	100	150	150	150	150		925	10,600	8.7
Attleboro	Mass.				125	125	150	150	150		700	10,810	6.4
Baraboo	Wis.	100	125	175	200	200	200	200	200		1400	10,810	13.4
Bay City	Mich.	50	75	100	100	125	125	125	125		825	6,760	12.3
Belleville	Ill.	125	150	250	250	250	250	250	250		1775	10,800	16.4
Berkeley	Cal.		50	200	220	200	200	200	200		1270	9,175	13.7
Beverly	Mass.	50	125	225	250	250	250	250	250		1650	10,400	15.8
Birmingham	Ala.		150	200	200	200	200	150			1100	8,155	13.4
Boone	Iowa	100	150	225	225	250	250	250	250		1700	10,800	15.7
Boston	Mass.	25	210	210	270	270	230	210	210		1635	10,570	15.4
Bowling Green	Ky.	50	70	75	100	100	125	125	125		770	10,650	7.2
Brockton	Mass.	120	180	210	240	270	270	270	210	210	1980	10,530	18.8
Burlington	Iowa	50	100	110	110	130	130	140	140		910	11,625	7.8
Cambridge	Mass.	75	150	200	250	250	225	250	300		1700	11,185	15.2
Canton	Ohio			125	125	150	175	200	200	200	1175	10,650	11.0
Chicago[1]	Ill.	150	200	250	150	150	150	150	105		1155	10,680	10.8
Chicago Heights	Ill.	75	100	150	150	150	150	150	150		1000	12,780	7.8
Cincinnati	Ohio	150	250	240	240	240	300	300	300		2080	11,070	18.8

[1] In the 8th grade time for 1st semester was 150 min. and for the 2d semester 60. The average of the two was taken.

TABLE XV — *Continued*

Cleveland . . .	Ohio	60	215	225	240	225	245	225	250		1685	10,900	15.4
Columbia . . .	S. C.	50	100	125	150	150	200	200			925	8,800	10.5
Columbus . . .	Ga.	200	250	280	300	300	300	300	300		1930	9,200	20.9
Cortland . . .	N. Y.	50	75	75	100	150	150	150	150		900	9,300	9.6
Covington . .	Ky.	125	300	300	300	300	300	300	300		2225	11,325	19.6
Crawfordsville .	Ind.		115	115	125	150	162	175			842	10,440	8.0
Creston	Iowa	50	100	100	125	125	125	150	150	225	1150	11,825	9.7
Danbury . . .	Conn.	100	100	150	125	125	150	125	125		875	8,100	10.7
Danville . . .	Ill.	50	50	100	100	125	125	150	150		900	8,550	10.5
Davenport . .	Iowa	75	150	160	175	125	150	150	150		1135	11,000	11.3
Decatur . . .	Ill.		50	200	200	200	250	250	200		1350	10,100	13.3
Detroit	Mich.	75	150	200	225	250	250	275	275		1775	11,090	16.0
Dover	N. H.	225	300	250	300	300	300	300	175		2150	11,000	19.5
Dubuque . . .	Iowa	50	75	100	125	150	150	150	150		950	10,725	9.8
		75	100	150	150	175	175	200	200		1225	10,745	11.4
Dunkirk . . .	N. Y.	100	125	300	300	300	300	225	225		1975	10,950	18.0
East Saginaw . .	Mich.	150	250	250	250	300	300	270			1770	10,600	16.6
Elgin	Ill.		50	50	100	100	100	125	125		650	9,600	6.7
Emporia . . .	Kan.	200	200	250	250	250	265	265	250		1930	10,800	17.8
Erie	Pa.	75	75	75	100	100	125	125	125		800	6,100	13.1
Fargo	N. Dak.		90	275	275	257	275	275	275		1740	11,550	15.0
Fort Smith . .	Ark.	125	250	300	300	300	300	300	300		2175	11,400	19.0
Fort Wayne . .	Ind.	100	100	125	150	150	150	150	150		925	10,800	8.5
Freeport . . .	Ill.	75	175	250	250	250	250	250	250		1750	11,4[illegible]5	15.2
Fulton	N. Y.	75	200	200	200	200	200	200	360		1625	9,550	17.9
Gloversville . .	N. Y.	100	150	150	200	150	175	225	225		1375	6,690	20.5
Grand Junction .	Colo.	75	100	125	125	125	125	125	150		950	9,200	10.3
Guthrie . . .	Okla.	50	170	250	275	270	250	255	275		1790	11,400	15.7
Hamilton . . .	Ohio	100	225	250	250	250	250	250	300		1875	10,400	18.0
Hannibal . . .	Mo.		75	100	100	125	125	125	125		775	11,000	7.4
Harrisburg . .	Pa.	150	150	150	225	225	225	225	225	225	1800	13,900	12.2
Houston . . .	Tex.	60	70	100	125	125	150	150	150		930	11,400	8.1
Huntington . .	W. Va.	100	150	200	200	200	200	200	200		1450	11,400	12.7
Hutchinson . .	Kan.	100	100	150	150	150	150	150	150		1100	8,800	12.6
Ironwood . . .	Mich.	75	100	100	125	150	150	240	240		1180	11,250	10.4
Ithaca	N. Y.		100	150	200	200	225	225	225		1325	12,000	11.0
Jersey City . .	N. J.	225	245	270	280	230	230	230	230		1940	11,000	17.6
Johnstown . .	N. Y.	150	200	200	200	200	200	275	300		1725	10,800	15.9

TABLE XV — *Continued*

City	State	Grade 1	2	3	4	5	6	7	8	9	Total Minutes per Week Arithmetic	Total Minutes per Week All Subjects	Percentage of Total
Joliet	Ill.	200	200	320	350	350	350	320	320		2410	12,200	19.7
Joplin	Mo.			100	100	125	150	150	150		775	12,660	6.1
Kalamazoo	Mich.	100	100	150	200	200	200	180	180		1360	11,050	12.3
Kansas City	Mo.	125	125	160	125	125	125	125	125		1035	11,455	9.0
Keokuk	Iowa	50	153	167	134	129	146	150	200		1129	11,000	10.2
Lancaster	Ohio	100	125	125	150	150	150	150	150		1100	9,930	11.0
Lansing	Mich.		150	150	150	150	150	150	150		900	10,500	8.5
Lewiston [1]	Maine	75	100	125	150	150	200	200	200	200	1400	12,906	11.1
Lincoln	Neb.	75	75	100	125	125	125	125	125		875	10,800	8.1
Long Beach	Cal.	45	60	100	125	125	125	125	125		830	10,200	8.2
Los Angeles	Cal.		60	100	100	100	100	90	90		640	10,375	6.1
Louisville	Ky.	50	75	125	125	125	150	150	150		950	10,195	9.3
Manchester	N. H.	75	245	300	300	300	250	250	290		2010	10,420	19.2
Manhattan	Kan.	100	100	125	125	150	150	150	150	150	1200	12,700	9.3
Mankato	Minn.	50	50	60	80	125	125	150	200		840	10,000	8.4
		60	60	75	100	150	150	175	200		970	10,000	9.7
Mason City	Iowa	100	150	200	225	225	225	250	275		1650	11,200	14.7
Memphis	Tenn.	75	100	125	215	100	100	100	90		815	11,200	7.2
Meriden	Conn.		150	200	225	250	250	250	250		1575	12,400	12.7
Milwaukee	Wis.	75	100	150	175	175	200	200	200		1275	12,100	15.1
Minneapolis	Minn.		150	200	200	225	225	180	180		1360	10,300	13.2
Muncie	Ind.		250	250	300	300	300	300	300		2020	11,100	18.2
Muscatine	Iowa		150	200	200	225	225	225	180		1460		
Muskogee	Okla.	100	150	200	250	250	250	300	300		1800	10,800	16.6
Nashville	Tenn.	75	75	150	150	150	150	150	150		1050	10,000	10.5
New Castle	Pa.	50	200	250	300	300	300	350	350		2100	9,400	22.3
New Haven	Conn.		75	75	100	100	100	125	150		725	11,200	6.4
New Orleans	La.	150	280	280	300	300	300	300	300		2210	10,800	20.4
Newport	Ky.	280	125	200	200	200	200	200	225		1555	10,275	15.1
Newton	Mass.	120	210	210	240	240	240	210	210		1780	10,110	17.6
New York	N. Y.	125	150	150	150	150	200	200	200		1325	9,888	13.4
Niles	Mich.	120	200	240	300	360	400	400	300		2320	10,400	22.3

[1] 75 in spring term of first year.

TABLE XV — *Continued*

Norfolk	Va.	200	200	225	225	250	250	250			1600	9,330	17.1
Norwich	Conn.	50	100	150	200	200	200	225	225		1350	10,925	12.3
Oakland	Cal.	150	150	300	300	300	300	180	180		1860	10,800	17.2
Oklahoma City	Okla.	50	75	100	100	125	125	150	150		875	9,200	9.5
Olean	N. Y.	50	200	250	250	250	250	250	225		1725	11,305	15.2
Paterson	N. J.	75	100	200	200	200	200	225	225		1425	10,600	13.4
Philadelphia	Pa.	150	200	200	225	225	225	225	225		1650	10,248	16.1
Phoenix	Ariz.		100	100	200	200	200	200	200		1200	10,800	11.1
Pittsburgh[1]	Pa.	60	120	180	200	200	240	300	360		1660	9,220	18.0
Pittsfield	Mass.		150	150	150	150	175	175	175		1125	10,400	10.2
Plainfield	N. J.		225	300	275	300	175	150	200		1525	9,200	16.5
Pomona	Cal.		75	100	100	125	125	125	125		775	10,550	7.5
Pontiac	Mich.			100	100	100	150	150	150		750	10,200	7.3
Port Huron	Mich.	50	75	75	100	100	100	125	125	200	950	12,665	7.5
Portland	Maine	75	200	250	250	250	250	200	200		1675	10,440	16.0
Providence	R. I.			200	200	250	250	250	250		1400	10,750	13.0
Reading	Pa.	200	280	300	250	250	250	250	250		2280	10,600	19.1
Richmond	Ind.		75	125	125	125	150	150	150		900	10,300	8.7
Riverside	Cal.			75	100	125	125	120	120		665	10,750	6.1
St. Cloud	Minn.	75	100	200	200	200	175	175	180		1205	10,800	11.1
St. Joseph	Mo.	75	100	100	125	125	150	150	165		990	10,600	9.3
St. Louis	Mo.	100	125	150	150	150	150	150	150				16.1
St. Paul	Minn.	100	150	200	250	250	250	250	250		1700		17.7
Salem	Oregon		75	100	100	100	125	150	150		800	10,200	7.8
San Francisco	Cal.	150	150	150	160	200	200	200	200		1410	8,120	17.3
Sault St. Marie	Mich.	50	75	75	100	100	100	125	125		750	10,600	7.0
Savannah	Ga.	300	350	400	450	450	400	350	375		3075	10,650	28.8
Schenectady	N. Y.	60	175	250	300	275	275	230	200		1765	10,930	16.1
Scranton	Pa.	150	210	210	290	290	290	300	300	300	2340	12,090	19.3
Sheboygan	Wis.	85	90	115	125	130	135	135	130	175	1120	11,045	10.0
Sioux City	Iowa		120	100	100	125	125	125	125		800	11,000	7.2
Sioux Falls	S. Dak.	50	100	200	250	300	300	300	300		1800	10,450	17.2
South Bend	Ind.		50	75	100	100	125	125	200		775	10,400	7.4
Spokane	Wash.	60	120	125	125	125	135	120	125		930	8,360	11.1
Springfield	Mass.		200	275	275	250	250	250	250		1750		
Superior	Wis.	100	125	175	200	200	200	200	200		1400	10,820	12.9

[1] Estimated.

TABLE XV — *Continued*

City	State	Grade 1	2	3	4	5	6	7	8	9	Total Minutes per Week Arithmetic	Total Minutes per Week All Subjects	Percentage of Total
Syracuse	N. Y.	70	150	150	150	150	150	180	180		1180	9,875	11.9
Tacoma	Wash.	25	100	225	275	275	275	275	250		1700	10,700	15.8
Tulsa	Okla.		150	200	200	200	200	250	250		1450	11,500	12.6
Vincennes	Ind.	75	75	115	125	140	140	140	140		810	11,400	7.0
Waterbury	Conn.	200	200	225	225	225	225	225	225		1750	12,000	14.5
Watertown	N. Y.	120	246	250	300	300	250	300	300		2066	10,625	19.4
Waterville	Maine	75	150	175	200	220	225	225	225	225	1500	11,100	13.5
Waukegan	Ill.		75	75	100	100	125	125	125		725	10,600	6.8
Wausau	Wis.	130	150	150	150	150	175	175	175		1255	11,300	11.1
Webb City	Mo.	125	125	125	150	150	150	175	175		1175	12,440	9.4
West Chester	Pa.	50	100	100	100	150	150	250	250		1150	9,600	11.9
Wilkesbarre	Pa.	60	120	120	150	150	150	300	300		1500	10,820	13.8
Winona	Minn.		70	75	100	100	100	125	150		720	10,200	7.0
Worcester	Mass.	100	225	240	250	260	250	250	250		1825		
York [1]	Pa.	150	300	300	300	300	300	300	300		2250	10,800	11.6
Youngstown	Ohio	100	125	125	150	150	150	150	150		1100	15,360	7.1

[1] Ad libitum.

TABLE XVI

ALL GRADES. DISTRIBUTION OF TIME SPENT ON ARITHMETIC IN MINUTES PER WEEK

TIME	CASES	TIME	CASES	TIME	CASES
640	1	1150	2	1750	3
650	1	1155	1	1765	1
665	1	1175	2	1770	1
700	1	1180	2	1775	2
720	1	1200	2	1780	1
725	2	1205	1	1790	1
750	2	1225	1	1800	3
770	1	1250	1	1820	1
775	4	1255	1	1825	1
800	3	1270	1	1860	1
810	1	1275	1	1875	1
815	1	1325	4	1930	2
825	1	1350	2	1940	1
830	1	1360	2	1950	1
840	1	1375	1	1975	1
842	1	1400	4	1980	1
875	3	1410	1	2010	1
900	4	1425	1	2020	1
910	1	1450	2	2025	1
925	3	1460	1	2066	1
930	2	1500	2	2080	1
950	4	1525	1	2100	1
955	1	1555	1	2150	1
970	1	1575	1	2175	1
975	1	1600	1	2225	1
990	1	1625	1	2250	1
1000	1	1635	1	2280	1
1035	1	1650	3	2255	1
1050	1	1660	1	2300	1
1100	4	1675	1	2320	1
1120	1	1685	1	2340	1
1125	1	1700	4	2410	1
1129	1	1725	2	3075	1
1135	1	1740	1		

The most noticeable feature of this table is the extreme variability that prevails among cities as to the

total amount of time per week given to arithmetic. The range is from 640 to 3075 minutes. No mode or central tendency is discoverable in the table. The same situation prevails when the facts presented in the foregoing table are brought together in closer formation by using a larger unit.

TABLE XVII

CLOSER DISTRIBUTION OF TIME ACCORDING TO MINUTES PER WEEK SPENT ON ARITHMETIC

Time	Cities
601– 700	4
701– 800	13
801– 900	13
901–1000	15
1001–1100	6
1101–1200	14
1201–1300	6
1301–1400	13
1401–1500	17
1501–1600	14
1601–1700	12
1701–1800	15
1801–1900	4
1901–2000	6
2001–2100	6
2101–2200	2
2201–2300	6
2301–2400	2
2401–2500	1
3001–3100	1
Total	148

Median — 1338 minutes per week Av. Dev. — 407.24 minutes
1st quartile — 950 3d quartile — 1746

These tables, showing a range of 2500 minutes and an average deviation of 407 minutes, clearly indicate the absence of a time standard.

NOTE. — The seventy-sixth case was taken as the median. The deviation for each group was found by finding the variation of the average of the group from the median and multiplying by the frequency.

The next two tables give the percentile distribution of the total school time devoted to arithmetic:

TABLE XVIII

Table Showing Distribution of Cities According to Per Cent of Total School Time, Exclusive of Recesses and Opening Exercises, Devoted to Arithmetic

Per Cent	Cases	Per Cent	Cases	Per Cent	Cases
6.1	3	11.0	3	16.0	2
6.4	2	11.1	5	16.1	3
6.7	1	11.3	1	16.4	1
6.8	1	11.4	1	16.5	2
7.0	3	11.6	1	16.6	2
7.1	1	11.9	2	17.1	1
7.2	3	12.2	1	17.2	2
7.3	2	12.3	3	17.3	1
7.4	2	12.5	1	17.6	2
7.5	1	12.6	1	17.7	1
7.8	3	12.7	2	17.8	2
8.0	2	12.9	1	17.9	1
8.1	2	13.0	1	18.0	3
8.2	1	13.1	1	18.2	1
8.4	1	13.2	1	18.8	2
8.5	2	13.3	1	19.0	1
9.0	1	13.4	4	19.1	1
9.3	3	13.5	1	19.2	1
9.4	1	13.7	2	19.3	1
9.5	1	13.8	1	19.4	1
9.6	1	14.5	3	19.5	1
9.7	2	14.7	1	19.6	1
9.8	1	15.0	1	19.7	1
10.0	1	15.1	2	20.4	1
10.2	1	15.2	3	20.5	2
10.3	1	15.4	2	20.7	1
10.4	1	15.7	2	20.9	1
10.5	3	15.8	2	22.3	2
10.7	1	15.9	1	28.8	1
10.8	3				

TABLE XIX

CLOSER DISTRIBUTION OF TIME ACCORDING TO PER CENT OF THE TOTAL RECITATION TIME DEVOTED TO ARITHMETIC

PERCENTAGES	CASES
6.1– 7	10
7.1– 8	14
8.1– 9	10
9.1–10	10
10.1–11	13
11.1–12	11
12.1–13	9
13.1–14	11
14.1–15	5
15.1–16	14
16.1–17	8
17.1–18	13
18.1–19	4
19.1–20	7
20.1–21	5
21.1–22	0
22.1–23	2
28.1–29	1
Total	147

Median — 12.7 % Av. Dev. — 3.69 %
1st quartile — 9.3 % 3d quartile — 16.5%

These tables show that about 13 per cent of the total school time is devoted to arithmetic. They do not tell us whether this is too much or too little. But if the cities in the lower quartile get satisfactory results with only 9 per cent of their total time given to arithmetic, then it is reasonable to suppose that all cities might reduce the time at least as far as the median.

NOTE. — The seventy-fourth case is taken as the median. The deviation for each group was found by finding the variation of the average of the group from the median and multiplying by the frequency.

NOTE. — There are only 147 cases in this table, since it was impossible to ascertain the total time spent in recitations other than arithmetic in one case.

CHAPTER IV

DOMINANCE OF METHODS IN THE TEACHING OF ARITHMETIC

The Topical vs. the Spiral Method

Methods in the teaching of arithmetic have been stressed differently at different times. Naturally the modes of instruction have been colored by the various stages through which the subject matter has evolved. Arithmetic grew by piecemeal; new topics, sections, or divisions were added as they were seen to bear a logical relation to old ones. Under the influence of scholars, the subject gradually assumed a highly unified and logical character. Later, however, it was observed that such an organization was not necessarily adapted to the most economical learning of the facts and processes of arithmetic. Consequently a reaction set in against the formal, adult, scientific attempts at the organization of the subject, and a readjustment of the materials to harmonize with the maturity levels of children began to receive increased attention. The older of these attempts at the organization of subject matter resulted in what is currently known as the topical method of

instruction; the younger, in what is currently known as the spiral method of instruction. By *topical method* is meant the presentation of topics sequentially related without any reference to the facility with which they may be learned, each topic being completed before the next one is presented. By *spiral method* is meant the presentation of recurring topics in widening concentric circles in harmony with the age and ability of the children being instructed. The topical method was presumed to give a notion of unity and continuity to the subject, while the spiral method was presumed to correspond to the psychological conditions of learning and to insure the fixation of the processes and skills of the subject.

Like every method, each of these, when it was not modified by the restraining and clarifying influence of the other, tended to swing to an unnatural extreme; this was particularly true when either was taught by inadequately trained or unsupervised teachers. We are not yet entirely rid of the baneful influence of these over-emphases. Although each in its time was an epoch-making method, reaction and readjustment were inevitable. As new topics were forced into the textbooks in arithmetic, the spirals were shortened, and the pupils were bewildered by the more frequent recurrence of the same topics. Textbook makers, catching the drift of this criticism as it came with ever increasing volume

from every section of the country, began to reduce the number of the spirals and to lengthen them.

The Spiral and Topical Methods in Different Geographical Regions

The present trend of educational sentiment with reference to the dominance of these two methods is shown in the tables that follow. These tables should be of service to textbook writers, to supervisors who are selecting textbooks and are planning courses of study, and to professionally inclined teachers who are interested in the drift of educational theory and practice.

TABLE XX

	Spiral Method	Topical Method	Combination of the Two	Total Number of Cities
North Central states	7	60	205	272
North Atlantic states	4	52	177	233
Western states	1	13	37	51
South Central states	3	16	50	69
South Atlantic states	0	6	22	68
Counties	1	26	68	95
Total	16	173	559	748

Table XX should be read as follows: 7 superintendents out of 272 in the North Central states favor the exclusive use of the spiral method, 60 favor the exclusive use of the topical method, while 205 favor a combination of the two.

TABLE XXI

(Table XX reduced to per cents)

	Spiral	Topical	Combined
North Central states	2.9	22.0	75.1
North Atlantic states	1.9	22.3	75.8
Western states	2.0	25.5	72.5
South Central states	4.0	23.2	72.4
South Atlantic states	0.1	21.4	78.6
Counties	1.0	27.4	71.6
	2.2	23.1	74.7

Table XXI is Table XX reduced to per cents.

Table XXI should be read as follows: 2.9 % of the superintendents in the North Central states favor the spiral method; 22 %, the topical method; and 75 %, the combination method. 2.2 % of all the superintendents irrespective of location favor the spiral method; 23 %, the topical method; and 75 %, the combination method.

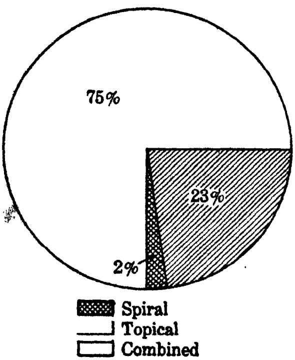

CHART VI.

The most striking feature of these tables is their uniformity. Clearly, opinion among school superintendents as to the desirability of either or both of these methods is well standardized. The number of superintendents advocating the spiral plan is almost negligible. Three fourths of the superintendents are of

the opinion that a combination of the two methods results in the most successful practice. The relative strength of the three groups is shown by Chart VI.

From the foregoing facts it would seem that which method shall prevail is no longer a mooted question. Sanity in combining the two methods is almost universally demanded by practitioners. For large geographical areas there is almost no perceptible variation in practice. With reference to this matter one section expects and demands what every other section expects and demands.

THE SPIRAL AND TOPICAL METHODS IN CITIES OF DIFFERENT SIZE

Although experience has been generalized for the different sections of the United States and for the United States as a whole, it remains to be seen whether there is any marked variation when these methods are distributed for different-sized cities. Not all of the superintendents who replied indicated the size of the city whose schools they superintend. The replies that were clear are distributed in the next two tables.

The data were distributed so as to show differences, if any, in practice in cities of ten different population classes. Cities in Rank I had a population of 1,000,000 or over; Rank II, 200,000 to 999,999; Rank III, 100,000 to 199,999; Rank IV, 50,000 to 99,999; Rank V, 30,000 to 49,999, etc.

TABLE XXII

Rank	City	Spiral	Topical	Combined	Total
I	1,000,000 and over	0	0	2	2
II	200,000 to 999,999	0	2	14	16
III	100,000 to 199,999	0	1	8	9
IV	50,000 to 99,999	1	7	23	31
V	30,000 to 49,999	0	8	32	40
VI	20,000 to 29,999	3	14	26	43
VII	15,000 to 19,999	1	12	31	44
VIII	10,000 to 14,999	1	16	68	85
IX	8,000 to 9,999	2	16	62	80
X	4,000 to 7,999	7	71	225	303
	Total	15	147	491	653

TABLE XXIII

(Table XXII reduced to per cents)

Rank	City	Spiral	Topical	Combined
I	1,000,000 and over	0	0	100.0
II	200,000 to 999,999	0	12.5	87.5
III	100,000 to 199,999	0	11.1	88.9
IV	50,000 to 99,999	3.2	22.6	74.2
V	30,000 to 49,999	0	20.0	80.0
VI	20,000 to 29,999	6.9	32.6	60.5
VII	15,000 to 19,999	2.3	27.3	70.4
VIII	10,000 to 14,999	0.2	19.8	80.0
IX	8,000 to 9,999	2.5	20.0	77.2
X	4,000 to 7,999	2.7	23.1	74.2
	All cities . . . . :	2.3	22.5	75.2

Table XXII shows that 2 superintendents in cities of 1,000,000 inhabitants or over favor a combination of the two methods; that 2 in cities between 200,000

and 1,000,000 prefer the topical, and 14 the combined methods; that 1 in a city between 125,000 and 250,000 prefers the spiral, 7 the topical method, and 23 the combined methods. Table XXIII presents the same facts reduced to per cents for each of the different-sized cities.

It is clear that the number contending for the spiral plan is too small to be worthy of serious consideration. It is true that there is a variation of opinion in nearly every sized city, except the very largest, but no significant conclusions may be drawn from this variation. The fact of most import shown by these tables is that the great majority of superintendents, regardless of the size of the cities, are agreed that a combination of the spiral and topical plans is better than either alone.

The Sanction of Usage

We are not here concerned directly with achievement or results, but with the opinions of school men as to the desirability of certain methods. Naturally the efficacy of the two methods is involved, but, so far as we know, no trustworthy tests have been made to determine this. It is doubtful if any are needed. The laborious processes of trial and error, of success and failure, under countless varying conditions, have sanctioned in no uncertain manner the discontinuance and elimination of the spiral

and topical plans as such, and have warranted the assumption that a combination of the two produces the best results. In the long run experience gained in this way is likely to be right. Until some one actually disproves that it is wrong or produces data that question its soundness, superintendents both young and old will be warranted in subscribing to those conditions and standards which their educational forbears have spent years in evolving. The weight of the testimony is so preponderantly in favor of a combination of the spiral and topical methods as to leave little room for doubt.

It is true that many questions relating to these two methods have been left unanswered. Our data do not show the exact manner in which this combination should be effected nor do they show how the material should be distributed grade by grade. Matters of such paramount importance as these have already attracted the attention of textbook makers, and there is scarcely a text of the last half dozen years that does not represent an attempt to make the proper adjustments. These are still matters of opinion that should be submitted to scientific scrutiny and investigation. But the men who are in the best position to do this are the superintendents themselves, and they are not likely to have the time or the inclination to do work of this character. Unless the experimental educationist comes to their rescue — and he is always likely to be handicapped by

remoteness from or unfamiliarity with actual schoolroom situations — we shall again "cut and try" through years of painful and more or less blind experimenting until by some happy turn of the wheel of fortune we shall arrive at a solution of our problem.

Methods of Subtraction

There are other phases of method concerning which superintendents have rather decided opinions. One of these relates to the manner in which subtraction shall be taught. Two methods are in vogue; one, the method of "taking from"; and the other, the so-called Austrian method of addition. Those who advocate teaching subtraction by "taking from" insist that there is no subtraction in the Austrian method, and those who advocate the Austrian method insist that it is far more psychological than the method of "taking from" because it does not involve a new mode of learning. The advocates of the Austrian method also contend that the skill acquired by its use will be more serviceable since it corresponds to the making change method employed by the business world. We are not now concerned with a presentation of the arguments of the two contending groups. Our problem is to determine the extent to which each of the devices has the sanction of usage. No doubt there are those who will maintain that usage is no measure of

the value of a tool. Such a criticism is entitled to consideration when the tool is used by an unintelligent or unskilled class of people; but when it is employed day after day and year after year with children of varying ages and circumstances by teachers and superintendents of reputed training and skill, such a criticism seems groundless. Surely the testimony of superintendents and teachers with reference to the intellectual instruments they daily use should be regarded as expert testimony.

Variation of Subtraction Methods in Different Geographical Areas

The following tables show that the Austrian method is universally regarded as second in importance, and that only a relatively small number and per cent of superintendents favor both methods.

TABLE XXIV

	Addition	Taking from	Both	All Cities
North Central	96	134	15	245
North Atlantic	105	107	8	220
Western	24	21	4	49
South Central	16	44	3	63
South Atlantic	11	14	1	26
Counties	12	47	3	82
Total	264	367	34	685

TABLE XXV

(The above table reduced to per cents)

	Addition	Taking from	Both
North Central	39.2	54.8	6.0
North Atlantic	47.7	48.6	3.7
Western	48.9	42.8	8.3
South Central	25.4	70.0	4.6
South Atlantic	42.3	54.0	3.7
Counties	39.0	57.3	3.7
Average	41.5	53.6	4.9

According to Table XXIV, 96 of 245 superintendents in the North Central states would teach subtraction by the Austrian method, 134 would teach subtraction by the "taking from" method, and 15 would use both methods. The same facts are presented in per cents in Table XXV.

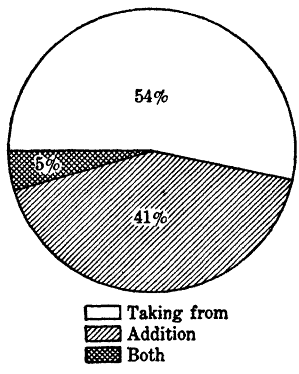

Chart VII.

Neither of the arrays indicates that there is a pronounced tendency in either direction. The surprising feature about the tables, although the range for both methods is from 3.7 per cent for county superintendents to 8.3 per cent for superintendents in the Western

states, is the agreement in the different geographical areas in regard to the undesirability of using both methods. For the country at large less than 5 per cent of the total number of superintendents is convinced of the value of using both methods. Here, as elsewhere in this material, we have fairly conclusive evidence that superintendents favor a specific way of doing things.

An examination of the percentile array shows that there is wide divergence of opinion among the different sections of the country. Seventy per cent of the superintendents in South Central states favors "taking from," while only 42.8 per cent of the superintendents in the Western states favors this method; this is a difference of 28.2 per cent. Such a difference cannot be accounted for by a difference in the mathematical necessities of the two sections of the country, for the practice of the South Atlantic states, as shown by the percentages, corresponds almost exactly to the central tendency of the country as a whole. It should be noted that the division of emphasis found in country schools closely corresponds with the division for the United States in general.

From the data thus far presented one cannot maintain that there is a marked tendency in either direction, nor can one safely predict what the practice of to-morrow will be.

Variation of Subtraction Methods in Cities

The data presented in the two preceding tables were redistributed to determine the variability of the use of these devices in different-sized cities.

TABLE XXVI

Cities	Addition	Taking from	Both	All Cities
1,000,000 and over	2	1	0	3
200,000 to 999,999	7	6	1	14
100,000 to 199,999	7	4	1	12
50,000 to 99,999	8	19	1	28
30,000 to 49,999	13	21	1	35
20,000 to 29,999	22	17	0	39
15,000 to 19,999	9	26	3	38
10,000 to 14,999	32	43	4	79
8,000 to 9,999	28	45	4	77
4,000 to 7,999	124	138	16	278
Total	252	320	31	603

TABLE XXVII

(Table above reduced to per cents)

Cities	Addition	Taking from	Both
1,000,000 and over	66.7	32.3	0.0
200,000 to 999,999	50.0	42.8	7.2
100,000 to 199,999	58.3	33.3	8.4
50,000 to 99,999	28.5	67.8	3.7
30,000 to 49,999	37.1	60.0	2.0
20,000 to 29,999	56.4	43.6	0.0
15,000 to 19,999	23.6	68.4	8.0
10,000 to 14,999	44.4	54.4	1.2
8,000 to 9,999	36.3	58.4	5.3
4,000 to 7,999	44.5	49.6	5.9
Averages	41.8	53.1	5.1

These tables show that the large cities favor the Austrian method, while the smaller cities favor the "taking from" method. It is admitted that educational progress usually takes root first in the large cities, and that tradition and conservatism are clung to more tenaciously in the rural and semi-rural districts. If these generalizations apply in this case, then we have here a positive tendency. It is true that a superintendent in a given sized city can by inspection determine whether he is to be counted with the majority or the minority, but he learns nothing from these tables as to what the tendency is, unless it be true that the practice of the large cities represents educational progress. He will know the extent to which his practice varies from current practice, and the remodification of emphasis that must be made in order to make his practice correspond more closely to the central tendency. Certainly superintendents in cities of Class I and those in Class VII, whose schools represent the extreme variations from the central tendency, need to justify their practice.

It is to be regretted that there are no conclusive results to submit regarding this phase of instruction. We merely have here another problem awaiting solution. Intelligent observation and experimentation are needed to determine which of the two methods is the more economical and which will produce the better results.

CHAPTER V

THE SEQUENCE OF THE MULTIPLICATION TABLES

The Influence of Tradition

Custom and convention have heretofore determined the order in which the multiplication tables have been taught. The custom has prevailed of teaching them in the order of the digits. This custom arose because it was presumed that the digits had originated in a 1, 2, 3 order. There is not the slightest evidence that this is true. So far as we have any evidence it tends to show that the digits were not invented in any regular order. It has long been a question with which school supervisors everywhere have been concerned, as to whether or not the present logical order of presentation is after all the most pedagogical. It is barely possible that the most difficult order of mastering the multiplication tables is the 1, 2, 3 order. Numerous experiments at varying this order have been tried, but, so far as we know, none of them bear the stamp of careful scientific work under controlled conditions. As yet we have no reliable information as to the best sequential arrangement of the tables for teaching purposes.

The Undermining of Tradition

Over 500 superintendents furnished us with testimony as to the order that they think is conducive to best results. Their replies are presented in the following tables:

TABLE XXVIII

Order of Teaching Multiplication Tables

	Regular Order	2, 4, 5, 10, etc.	No Tables but Combinations	Not Important	Misc.
North Central	58	57	5	15	51
North Atlantic	58	49	18	9	29
Western	14	5	2	1	6
South Central	17	7	5	4	8
South Atlantic	9	5	0	1	2
Counties	39	9	3	8	16
Total	195	132	33	38	112

TABLE XXIX

(The above table reduced to per cents)

	Regular Order	2, 4, 5, 10, etc.	No Tables but Combinations	Not Important	Misc.
North Central	31.2	30.7	2.7	8.1	27.3
North Atlantic	33.5	28.3	10.4	5.2	22.6
Western	50.0	17.8	7.2	3.6	21.5
South Central	41.5	17.1	12.3	9.7	19.9
South Atlantic	53.0	29.7	0.0	5.8	11.5
Counties	52.0	12.0	4.0	10.7	21.3
Average	37.5	25.4	6.4	7.3	23.4

According to Table XXVIII, out of 186 superintendents in the North Central states 58 maintain that the multiplication tables should be taught in the regular order, 57 that they should be taught in a 2, 4, 5, 10 order, 5 that the combinations should be presented without any reference to order, 15 that an order is of no importance whatever, and 51 that any miscellaneous order will be satisfactory. The distribution of the replies from the other geographical divisions may be read in the same way. Table XXIX is Table XXVIII reduced to per cents.

The second column in each of these tables — the one headed 2, 4, 5, 10 — is unsatisfactory, for the reason that the 132 superintendents who insisted upon this order did not agree as to the order that should prevail for the other tables. Some said 3, 6, 7, 8, 9; others, 8, 3, 6, 7, 9; still others, 8, 3, 9, 7, 6; still others, 8, 3, 6, 9, 7; every possible combination, in fact, was presented.

It will be noted from these tables that a majority of the superintendents are of the opinion that the regular order is not the best order. Thirty-three or 6.4 per cent of the superintendents insist that no tables at all should be given, — that the combinations should be presented without reference to any systematic arrangement. Thirty-eight or 7.3 per cent insist that the order of the tables is not a matter of importance, and 23.4 per cent believe that a miscellaneous presentation of them

insures the most satisfactory results. Only 37.5 per cent cling to the traditional order.

Chart VIII shows roughly that four superintendents in every ten are satisfied with the regular order and that a little over one half are not in agreement as to the order that should be used.

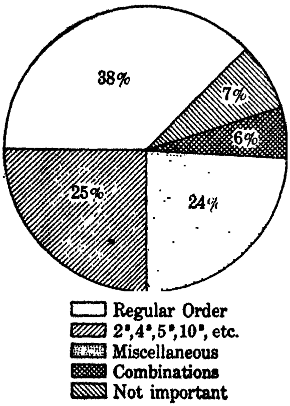

CHART VIII.

An inspection of the variation of the different geographical regions as indicated by the first column of Table XXIX suggests the lack of agreement. The range is from 31.2 per cent in the Central states to 53 per cent in the South Atlantic states. The tables indicate that the North Central states are least disposed to cling to tradition and that the South Atlantic states are most conservative in this matter.

The difference between the extremes in the miscellaneous column — the North Central and the South Atlantic states — is 16.8 per cent. The divergences in the other cases are so marked as to indicate that much experimenting needs to be done before we shall arrive at a final solution of this important problem. Undoubtedly we are warranted in concluding that we have evidence here of a decided tendency to reconstruct a mode of instruction.

If a number of coöperative movements could be established and different orders were tried simultaneously in a number of places and the results carefully checked by some uniform system, we might hope to determine at an early date upon the best order of presentation.

We have distributed this material on the basis of the different-sized cities from which the data were collected.

TABLE XXX

ORDER OF TEACHING MULTIPLICATION TABLES

Population (By size of city)	Regular Order	2, 4, 5, 10, etc.	No Tables	Not Important	All Cities
1,000,000	1	1	0	0	2
200,000–999,999 . . .	4	3	2	2	13
100,000–199,999 . . .	2	1	2	1	6
50,000– 99,999 . . .	6	5	4	5	22
30,000– 49,999 . . .	11	8	3	4	26
20,000– 29,999 . . .	10	5	3	6	26
15,000– 19,999 . . .	10	10	2	5	30
10,000– 14,999 . . .	20	22	2	11	58
8,000– 9,999 . . .	29	17	0	9	57
4,000– 7,999 . . .	63	51	12	53	195

Table XXX gives the number of superintendents replying to each question for each of the different-sized cities, and Table XXXI gives the same facts reduced to per cents. It should be noted that there are ten population classes, later referred to as Class I to Class X, from largest to smallest.

TABLE XXXI

(Above table reduced to per cents)

	Regular Order	2, 4, 5, 10, etc.	Combinations	Not Important	All Cities
1,000,000	50.0	50.0	0.0	0.0	0.0
200,000–999,999 . . .	30.8	23.0	15.4	15.4	15.4
100,000–199,999 . . .	33.3	16.7	0.0	33.3	16.7
50,000– 99,999 . . .	27.3	22.7	9.1	18.2	22.7
30,000– 49,999 . . .	42.3	30.7	0.0	11.6	23.1
20,000– 29,999 . . .	38.4	19.2	7.7	11.6	23.1
15,000– 19,999 . . .	33.3	33.3	10.0	6.7	23.1
10,000– 14,999 . . .	34.5	37.9	5.2	3.5	18.9
8,000– 9,999 . . .	50.9	30.0	5.3	0.0	15.8
4,000– 7,999 . . .	32.3	26.2	8.2	6.1	27.2
Average . . .	35.9	28.3	6.9	22	22

These tables are more instructive than the preceding tables. The two superintendents from cities of over a million inhabitants are evenly divided upon the advisability of clinging to the regular order, but, with the exception of these and those of Class IX, nearly two thirds of the superintendents in the other cities are agreed that we need a reorganization and readjustment of the customary order employed in teaching the multiplication tables.

The distinguishing characteristic of any single column is its variability. For example, only 16.7 per cent of the superintendents in cities ranging from 100,000 to 199,999 believe in the 2, 4, 5, 10 order, while 37.9 per cent of those in cities between 10,000 and 14,999 and 50 per cent of

those in cities of 1,000,000 or over advocate this order. None of the superintendents in cities of Class I, III, or V refer to teaching multiplication wholly by combinations, while 10 per cent of those in Class VII and 15.4 per cent of those in Class II urge this plan. Combining the replies of those listed in columns four and five, those who would insist upon some order but who are not concerned about any exact order, we have a variation ranging from 15.8 per cent in cities of 8000 to 9999 to 50 per cent in cities of 100,000 to 199,999.

No decided tendency can be detected in any of the columns nor in any of the different-sized cities, unless it be the general dissatisfaction that seems to prevail with the existing mode of procedure. Apparently superintendents are still at sea with reference to this phase of technique.

The Effect of Educational Dissatisfaction

An examination of such tables as these tends to intensify the general feeling of unrest that these tables reveal. The prevalence of a widespread dissatisfaction with the regular order of teaching the multiplication tables is, we believe, healthy and will in the long run result in more economical instruction.

A superintendent may utilize the tables to discover the particular group of reactionaries or conservatives to which he belongs. For it will be observed that super-

intendents in Class IV apparently are not so conservative as those in Class IX, while those in Class IV are more radical than those in Class I. After all, the particular group to which one belongs is a relative matter.

School supervisors of every degree of experience will find many questions in this part of this investigation. Those who have reached a final conclusion regarding this whole matter, if there be any such, should give their results to the world so that the uninitiated may not stumble into the pitfalls of tradition. On the basis of our returns, it seems that sound advice cannot be given to a young superintendent. The only conclusion to which we can come is that the old order is still the prevailing order, and that the prevailing tendency is to try to find some other.

CHAPTER VI

ORAL WORK IN ARITHMETIC

While it is true from one point of view to state that all teaching in the elementary grades is of an oral character, yet in recent years there has come to be a more or less sharp differentiation between the oral and other types of work. It is stated in many courses of study that work of a certain grade is to be treated orally, or that a certain bit of subject matter is to be treated orally. Consequently it is very difficult to make a true interpretation of any statements relating to the amount of oral work done in any subject.

The authors asked the superintendents in the cities throughout the country in towns of four thousand and over, to state "the per cent of recitation time in each grade which should be given to oral work in each grade."

	Grade								
	1	2	3	4	5	6	7	8	9
Per cent	—	—	—	—	—	—	—	—	—

In view of the whole schedule, the authors have deemed it safe to interpret these results according to their face

value. The replies from one half of the superintendents throughout the country are tabulated in Table XXXII, so as to show the median per cent of recitation time proposed for oral work in arithmetic in the different geographical sections.

TABLE XXXII

MEDIAN PER CENT OF RECITATION TIME PROPOSED FOR ORAL WORK IN ARITHMETIC

GEOGRAPHICAL DIVISIONS	I	II	III	IV	V	VI	VII	VIII
North Central .	66.0	68.0	49.0	39.0	32.0	29.0	18.0	17.0
North Atlantic .	47.5	50.0	42.0	31.3	29.6	25.0	19.0	15.2
Western . . .	55.0	60.0	52.5	35.0	30.5	32.0	27.6	20.0
South Central .	54.0	44.0	35.0	24.6	19.0	16.0	13.0	9.0
South Atlantic .	54.0	42.5	32.0	28.2	17.3	15.2	10.2	10.2
	55.3	52.9	42.1	31.6	25.7	23.4	17.6	14.3

The table should be read as follows: In the North Central territory the median per cent of recitation time devoted to the oral work in arithmetic is 66 per cent in the first grade; in the North Atlantic territory, 47.5 per cent; in the Western territory, 55 per cent; in the South Atlantic territory, 54 per cent. The average of the medians for the country as a whole for the first grade is 55.3 per cent. It will be observed from an examination of this table that there is a regular and fairly gradual reduction in the median amount of time given to oral work with each succeeding grade. This no doubt is what we should expect. As students in-

crease in maturity and in facility they should grow more and more able to work independently of the teacher. One of the measures of the effectiveness of instruction is the extent to which pupils can work independently of the teacher in those subjects in which they have been instructed for a long time.

Attention is directed to the fact that the median time given to the oral recitation work in arithmetic in the first, second, fourth, and fifth grades is high in the North Central states. The Western states show the high per cent of recitation time given to oral work in arithmetic in the third, sixth, seventh, and eighth grades. However, these variations are not great.

In order to ascertain whether differences in regard to the median per cent of time given to the oral work in arithmetic exist in different-sized cities, the following table was prepared:

TABLE XXXIII

MEDIAN PER CENT OF RECITATION TIME PROPOSED FOR ORAL WORK IN ARITHMETIC

SIZE OF CITY	I	II	III	IV	V	VI	VII	VIII
100,000 and over	57.	49.	43.	32.	27.	23.	18.	14.
30,000 to 100,000	42.	46.	40.	28.	25.	21.	13.	12.
15,000 to 30,000	49.	64.	47.	33.	25.	24.	21.	19.
10,000 to 15,000	34.	44.	34.	27.	24.	23.	16.	12.
8,000 to 10,000	26.	51.	33.	29.	26.	20.	17.	12.
4,000 to 8,000	23.	50.	36.	33.	27.	27.	25.	29.
Average	39.	51.	39.	30.	26.	23.	18.	16.

This table should be read as follows: In cities of 100,000 or over the median per cent of recitation time devoted to oral work in arithmetic in the first grade is 57 per cent; in cities between 30,000 and 100,000, 42 per cent; between 15,000 and 30,000, 49 per cent; between 10,000 and 15,000, 34 per cent; between 8000 and 10,000, 26 per cent; between 4000 and 8000, 23 per cent; and the average of the country as a whole being 39 per cent. It will be noted that the median per cent of recitation time given to oral work in arithmetic is highest in the first grade. The highest median per cent of recitation time for the second and third grades is found in cities of from 15,000 to 30,000 population. No sharp lines of distinction can be drawn for grades four, five, and six. The median per cent of recitation time given to oral work in the seventh and eighth grades is greater in the smaller cities. It seems that the larger the city the greater the amount of time given in the lower grades to oral work and the more constant is the reduction of time grade by grade, while the smaller the city the more uniform is the amount of time given to oral work grade by grade.

While it is true that these data may not be absolutely accurate, yet in consideration of the fact that these replies were received from cities of varying sizes, located in different parts of the country, we believe that they are fairly reliable, and of importance to the supervisor of the teaching of arithmetic.

The opportunities for waste in connection with the oral treatment of a subject like arithmetic are great. It is desirable for the supervisor to know what is going on during this period of oral work. In the first three grades about one half of the recitation time is assigned to this type of activity; in the intermediate grade about one third; in the grammar grades from one sixth to one seventh. Hence, there are numerous opportunities for wastefulness in the recitation throughout these different grades.

CHAPTER VII

DRILL IN ARITHMETIC

Much interest attaches to the general problem of the proportion of time which should be given over to drill in teaching arithmetic. Many of the theories of modern education have been such as to discourage teachers in the matter of drill, and to encourage them to emphasize rationalization of the arithmetical work. This has led many teachers to teach the technique of arithmetic inadequately.

While it is true that no conclusive answers have been arrived at by the educational experimentalist, yet it is nevertheless important to note that the experiments which have been made with a view toward determining the value of drill have tended toward the conclusion that drill within certain limits is of distinct educational value. Such studies as those of Mr. J. C. Brown and Dr. T. J. Kirby (referred to in a later chapter) indicate clearly that short rapid drill is of importance. Dr. Kirby experimented with a group of children in the upper grades, wherein an opportunity was given for evaluating the effectiveness of drill periods of different lengths. His conclusions were that much might be gained through the introduction of a short drill period in every arithmetic recitation.

In a recent investigation conducted by the authors

data concerning the per cent of recitation time which should be given to drill were received from 564 cities. These data were distributed for the different sections of the United States and also for the cities of different size for the purpose of discovering the variations in practice in each grade. The table below shows this variation.

TABLE XXXIV

THE MEDIAN PER CENT OF RECITATION TIME FAVORED FOR STRICTLY DRILL WORK BY 564 SUPERINTENDENTS DISTRIBUTED THROUGHOUT THE DIFFERENT SECTIONS OF THE COUNTRY

GRADE	I	II	III	IV	V	VI	VII	VIII
North Atlantic	49	60	69	45	42	32	24	18
South Atlantic	37	44	40	31	32	19	17	12
North Central	29	53	52	46	38	28	21	17
South Central	45	46	45	42	35	28	23	12
Western	26	48	52	52	42	45	28	21
United States as a whole	43	50	52	45	39	31	22	17

The table should be read thus: The median per cent of recitation time favored for strictly drill work in the North Atlantic section is 49 for the first grade, 60 for the second grade, 69 for the third grade, etc.

It may be seen that there is a disposition to give a much smaller proportion of the time to drill in the upper grades than in the lower grades. Though there is a considerable variation in the different sections of the country, the general emphasis is quite similar.

The table below shows the data distributed for the cities of different size.

TABLE XXXV

MEDIAN PER CENT OF RECITATION TIME FAVORED FOR DRILL BY SUPERINTENDENTS IN CITIES OF DIFFERENT SIZE

Cities	Grade							
	I	II	III	IV	V	VI	VII	VIII
100,000 and over	33	46	57	51	41	31	29	18
30,000 to 100,000	25	29	50	44	35	32	21	15
15,000 to 30,000	38	51	58	45	37	29	19	17
10,000 to 15,000	31	52	57	48	39	31	22	16
8,000 to 10,000	42	53	52	42	36	27	21	16
4,000 to 8,000	44	53	50	45	36	21	22	17

This table should be read thus: The median per cent of recitation time favored for strictly drill work in cities of 100,000 and over for the first grade is 33, for the second grade 46, for the third grade 57, etc.

It is interesting to note that there seems to be no striking differences in the attitude of the superintendents in large cities or in small cities.

Chart IX shows the attitude of superintendents toward drill work in the recitations in each grade in the 564 cities. The upper line shows that 75 percentile, that is to say, one fourth of the superintendents favor the percentage of time indicated or more for each grade. For example, one fourth of the superintendents favor the giving of 79 per cent or more of the time to strictly drill work in the first grade; 81 per cent or more to strictly drill work in the second grade; 74 per cent or more to strictly drill work in the third grade; 64 per cent or more to strictly drill work in the fifth grade; and so on. The middle line represents the median, which

means that one half of the superintendents favor giving 43 per cent or more of time in the first grade to drill work; 50 per cent or more in the second grade; 22 per cent or more in the third grade; 26 per cent or more in the eighth grade. The lower line is the lower quartile, and shows that one fourth of the superintendents are in favor of giving 24 per cent or less of the time to drill work in the first grade; 27 per cent or less in the second grade; 34 per cent or less in the third grade; 31 per cent or less in the fourth grade; and 6 per cent or less in the eighth grade.

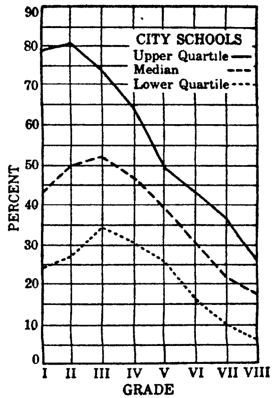

CHART IX. Per cent of recitation time to drill.

Chart IX shows the median per cent of recitation proposed for strictly drill work in cities of different size throughout the country. The close agreement is striking. Generalizations based on such wide experience are of importance in arriving at any satisfactory solution of the problem. The fact that the curves descend after the third grade is of special interest.

In the absence of experimental knowledge as to the exact per cent of time which should be given to drill, the supervisor is interested in knowing what the attitude of other supervisors may be in regard to this policy. While we have no adequate means of evaluating the comparative

arithmetical efficiency obtained in the schools differing in the amount of time given over to drill, it is important that the amateur know the prevalent practice. It would seem safe to say that if one fourth of the cities are devoting 80 per cent or more of the recitation time in the third grade to drill; 50 per cent or more of the recitation time in the fifth grade; and 25 per cent or more of the recitation time in the eighth grade, we should inquire into the effectiveness of the work in the 140 cities devoting 35 per cent or less in the fifth grade, decreasing to 5 per cent or less in the eighth grade. In other words, such wide variation suggests the probability of waste in the method at this point.

A thoroughgoing comparison of results attained in cities with widely different standards for the time given to drill would be of great value if constructive recommendations are to be made. However, in view of the results of the detailed investigations on the value of drill in arithmetic which have thus far been made, we are certain that short drill periods produce the best results.

In the absence of more satisfactory data for the evaluation of the different degrees of emphasis on drill, it is of importance to know the central tendency of practice, based on the experience in hundreds of cities. No doubt many supervisors will wish to adopt the median for each grade proposed above by the superintendents as the most satisfactory time limit. (Chart IX shows this clearly.)

Note. For more detailed information concerning the amount of time given to drill work see Appendix B.

CHAPTER VIII

GRADE FOR INTRODUCTION OF TEXT IN ARITHMETIC

THE supervisor who has been interested in the practice of his neighbor with reference to the introduction of a textbook in arithmetic has no doubt been impressed with the fact that wide variations exist in this particular. Any investigation covering a small number of cities presents such wide variations as to make it impossible to form an intelligent opinion as to the practice in this connection. With data from hundreds of superintendents distributed throughout the country, however, it is possible to make certain generalizations with regard to the prevailing practice. The writers, in connection with the report of the N. E. A. Committee on Economy of Time in Arithmetic, received replies from 754 cities bearing upon this problem. General surveys of this sort are both interesting and valuable. They show the spread of practice and they reveal the standardization of opinion. The data collected from the 754 cities are presented in the following tables:

TABLE XXXVI

SHOWING GRADE IN WHICH AN ARITHMETIC TEXT IS INTRODUCED

BY GEOGRAPHICAL DIVISIONS	I	II	III	IV	V	VI	TOTAL
North Central	1	18	160	64	22	2	267
North Atlantic	2	9	122	78	15	0	226
Western	0	6	25	17	2	0	50
South Central	0	13	44	17	1	0	75
South Atlantic	0	4	19	5	2	0	30
Counties	2	16	53	28	4	3	106
	5	66	423	209	46	5	754

The meaning of this table becomes clear when read as follows: Of the 267 cities reporting from the North Central territory, 1 introduced a text in the first grade; 18, in the second grade; 160, in the third grade; 64, in the fourth grade; 22, in the fifth grade; and 2 in the sixth grade. Again, of the five cities introducing a textbook in the first grade, 1 is in the North Central territory; 2, in the North Atlantic territory; and 2, in the country schools reported by the county superintendents. Of the 66 schools introducing a textbook in the second grade 18 are in the North Central territory; 9, in the North Atlantic territory; 6, in the Western territory; 13, in the South Central territory; 4, in the South Atlantic territory; and 16, in the counties reported by the county superintendents. Attention is directed to the variation represented by isolated cases; for example, 5 superintendents introduce

a textbook in the first grade while 5 other superintendents do not introduce a textbook until the sixth grade. Again, the 66 superintendents who introduce a textbook in the second grade disagree in policy and practice with the 46 superintendents who introduce a textbook in the fifth grade. However, despite this variation, it is of significance to note that the prevailing tendency is to introduce a textbook in the third grade or the fourth grade; thus experience seems to point to these as the standard grades for the introduction of a textbook. (It should be noted that the distribution resembles the distribution to be expected by chance.)

The following table shows the same facts reduced to per cents. The third and fourth grades are even more clearly shown to be the dominant grades for the introduction of a textbook. Almost 85 per cent of the cities introduce a textbook in one or the other of these grades. There seems to be no striking differences due to geographical location, the third grade being the modal grade in each section of the country, and the fourth grade standing second in each section of the country. About the same percentage of superintendents wait until the fifth grade to introduce the text as introduce it in the second grade, and exactly the same percentage wait until the sixth as introduce it in the first grade.

TABLE XXXVII

(Preceding table reduced to per cents)

	I	II	III	IV	V	VI	TOTAL
North Central	0.3	6.8	60.0	24.0	8.2	0.7	100
North Atlantic	0.9	4.2	53.8	34.4	6.7	0.0	100
Western	0.0	12.0	50.0	34.0	4.0	0.0	100
South Central	0.0	17.3	58.7	22.7	1.3	0.0	100
South Atlantic	0.0	13.3	63.3	16.7	6.7	0.0	100
Counties	1.9	15.2	50.0	26.4	3.7	2.8	100
Average	0.7	8.7	56.1	27.7	6.1	0.7	100

The meaning of this table becomes clear when read as follows: In the North Central territory .3 per cent of the schools introduce a textbook in the first grade; 6.8 per cent, in the second grade; 60 per cent, in the third grade; 24 per cent, in the fourth grade; 8.2 per cent, in the fifth grade; and .7 per cent, in the sixth grade.

In the absence of striking sectional differences, the question arises as to whether or not differences in the year in which a textbook is introduced may not be correlated with the size of the city; that is, are textbooks in arithmetic introduced earlier or later in large cities than in small cities? If one were to hazard an opinion on the basis of the foregoing facts, he would infer that they are introduced earlier in the smaller places. The following table shows how nearly current such an opinion is.

TABLE XXXVIII

SHOWING GRADES IN WHICH ARITHMETIC TEXT IS INTRODUCED

POPULATION (By size of city)	I	II	III	IV	V	VI	TOTAL
1,000,000 and over	0	0	1	0	0	0	1
200,000 to 999,999	0	1	9	5	0	0	15
100,000 to 199,999	0	0	8	4	1	0	13
50,000 to 99,999	0	2	20	8	1	0	31
30,000 to 49,999	0	2	22	12	3	0	39
20,000 to 29,999	1	1	26	11	3	0	42
15,000 to 19,999	0	1	30	9	1	0	41
10,000 to 14,999	0	10	45	26	7	0	88
8,000 to 9,999	0	4	41	25	7	2	79
4,000 to 7,999	2	29	168	81	19	0	299
	3	50	370	181	42	2	648

The meaning of this table becomes clear when read as follows: In the one city of 1,000,000 and over reporting, textbooks are introduced in the third grade. Of the fifteen cities of 200,000 to 999,999 reporting, 1 introduces a text in arithmetic in the second grade; 9, in the third grade; 5, in the fourth grade.

It is interesting to note that the greater variations appear in the smaller cities. All the cities introducing arithmetic in the first grade are in towns with a population of 30,000 or less. Four fifths of the cities introducing a textbook in arithmetic in the second grade are in cities of 15,000 or less. Three fourths of the cities introducing a textbook in arithmetic in the fifth grade are in towns of 15,000 or less. The variation is revealed even more clearly in the table of percentages below.

TABLE XXXIX

(Preceding table reduced to per cents)

Population	I	II	III	IV	V	VI	Total
1,000,000 and over	0.0	0.0	100.0	0.0	0.0	0.0	100
200,000 to 999,999	0.0	6.7	60.0	33.3	0.0	0.0	100
100,000 to 199,999	0.0	0.0	61.5	30.7	7.6	0.0	100
50,000 to 99,999	0.0	6.5	64.5	25.8	3.2	0.0	100
30,000 to 49,999	0.0	5.2	56.4	30.7	7.7	0.0	100
20,000 to 29,999	2.4	2.4	61.8	26.2	7.2	0.0	100
15,000 to 19,999	0.0	2.5	73.2	21.8	2.5	0.0	100
10,000 to 14,999	0.0	11.3	51.1	30.0	7.6	0.0	100
8,000 to 9,999	0.0	5.2	51.9	31.7	8.7	2.5	100
4,000 to 7,999	0.8	9.6	56.2	27.1	6.3	0.0	100
	0.6	7.7	57.1	28.0	6.2	0.4	100

The meaning of this table becomes clear when read as follows: In cities of 1,000,000 population or over, 100 per cent introduce a textbook in the third grade; in cities of 200,000 to 999,999 population, 6.7 per cent introduce a textbook in the second grade; 60 per cent in the third grade; 33.3 per cent in the fourth grade, etc. Here again it is clear that experience has been standardized in cities of every size; the third grade is the modal grade for the introduction of a textbook. Chart X presents this conclusion graphically.

From the foregoing presentation of replies from superintendents distributed throughout the United States, and in cities of different size, we are justified in the following conclusions: A superintendent who introduces

a textbook as early as the first or second grade, or who postpones the introduction of such text as late as the fifth or sixth grade, will do so in the face of generalized practice. While we do not know as the result of careful investigation the best time to introduce a textbook in arithmetic, we do know that in the experience of the thousands of teachers and of the hundreds of superintendents represented in this study the third grade is the best grade for the introduction of this subject, with the fourth grade standing second.

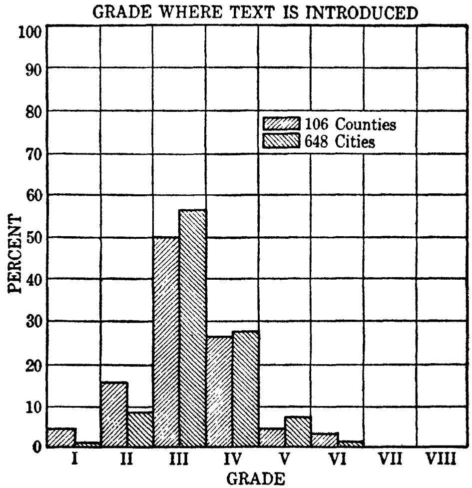

CHART X.

It would be of great administrative importance for us to know about the results obtained in arithmetic work secured in a school which postpones the introduction of a textbook until the fifth or sixth grade. From an investigation of isolated instances, where the textbooks have been introduced very late, we have reason to believe that much of the arith-

metic work which is commonly associated with the textbook is really taught in the earlier grades. In other words, the extreme postponement indicated in the foregoing table in all probability represents an attempt to get away from the use of the textbook, rather than an attempt to get away from the actual teaching of arithmetic.

Again, students of this problem are concerned with the question as to which is the better grade for the introduction of a text, the third or the fourth grade. This can only be determined by careful tests, but the amount of time to be saved is of sufficient importance to justify the attempt to determine which practice is the better. This experimentation is going on, as is shown in the foregoing tables. What is needed now is a thoroughgoing coöperative investigation of results obtained under the different systems.

The advocates of the policy of concentration of the energy of the school toward the mastery of reading in the first three grades may find much to encourage them in this report. If a third of the schools are already postponing the introduction of a textbook in arithmetic until the fourth grade, there need be little difficulty in getting more time for reading during the first three grades.

Students of educational administration who take cognizance of the wide variation in age and maturity

of children in a particular grade may be led to question the advisability of postponing formal instruction in arithmetic to the upper grades. Again, the student who is conscious of the enormous amount of elimination which goes on in the early grades may question the policy of allowing children to postpone the introduction of a textbook in arithmetic until so late in their scholastic career.

CHAPTER IX

JUDGING TEXTBOOKS

SOCIETY through legislation fixes the curriculum of the school. Legal enactments do not go so far, however, as to prescribe the topics within the field of a particular subject. That task is left to the school officers. The schoolmaster usually is assigned the responsibility of selecting the particular topics within the various fields of subject matter, and of determining the necessary time and sequence to be given them.

THE TEXTBOOK AS A BASIS FOR THE COURSE OF STUDY

An analysis of courses of study published throughout the country by different school boards indicates that the particular topics taught, with their sequence and time allotments, are determined largely by the nature of the textbook in use. The most common statement found in a course of study is that the fifth grade is to take the material found in the adopted text from pages 120 to 160 in a given period. Thus, the school officials leave the choice and arrangement of particular topics

to the textbook writers. Indeed, it may be said that textbook writers in America are the ones who determine not only the choice and sequence of topics but also their gradation.

In view of this dependence upon the textbook, it is important for us to know that textbooks vary in all three of the foregoing features. Some textbooks ignore completely certain topics which are heavily stressed in others. The sequence of topics varies widely. The gradation likewise presents astounding differences. If the school officials desire to maintain that a text with 300 pages, which is to be used in the fifth and sixth grade, is to be divided so that 150 pages are to be taught in the fifth grade, 150 pages in the sixth grade, and that in turn 75 pages are to be taught in the first semester, 75 pages in the second semester, 25 pages in the first third of the first semester, and so on, they should be assured that the textbook is so graded that the children will find the units of 25 pages for a six weeks' period to be of equal difficulty. But we do not have such assurance. If we are to adapt textbooks to this scheme of supervision, it will be necessary for textbook writers to grade their material more scientifically. No doubt reliance upon the author's selection of topics with their sequence and gradation has been responsible for a large share of the unsatisfactory results in teaching.

Principles Underlying Course of Study Making

In arithmetic it is of especial importance that the supervisor or teacher select each topic with the greatest care, with a clear knowledge of the social, economic, and psychological features involved. It is likewise necessary that the sequence with which these topics are taught should be determined by the psychology involved rather than by the logic contained in the subject matter. As was indicated in an earlier chapter, there is a clear tendency toward breaking away from the supposed sequences in arithmetic. The teaching of the multiplication tables in the one, two, three order was based on the supposed logic of the case. Many teachers have found it economical, however, to present the material in a two, four, five, ten order. Again, certain theoretical considerations have suggested the desirability of postponing decimal fractions until after the completion of common fractions. However, it is a fact, that children who have completed a study of United States money are able to make a study of decimal fractions without the knowledge of common fractions. Other instances of a similar character abound. The supervisor who is intent upon securing maximum results is no longer satisfied to trust the sequence of topics within a given textbook. Rather will he be interested in establishing a sequence of topics on the basis of the child's difficulty

in mastering them and in light of the social need of the topics.

The theory of the graded courses of study is that the work will be adapted to the different stages of mental maturity. To do this is a difficult task and requires keen analysis, experimentation, and constant attention. Every supervisor and teacher must recognize the variation in ability that exists between classes within the same grade and adjust the materials accordingly. If the teacher rests content after presenting the material according to the plan of the textbook, there need be no surprise if the results are unsatisfactory. Every wide-awake teacher has recognized the fact that equal page allotments of material are of unequal difficulty.

Reliance upon Textbooks

German and French teachers do not rely upon the textbook as heavily as American teachers. Foreign teachers present their material fresh to the children from day to day, grading it as it is given to the needs of the particular class under instruction. Whenever we have teachers who know the quantitative demands of social and industrial life, who know arithmetic as a science, and who know children and their individual difficulties, we shall have arithmetic taught more effectively, and with far less effort than is being expended at the present time.

Changes in text are not always made because the new book is better than the old book, but because the old book has been in use for a considerable number of years. It is the fashion to change books periodically, and adult human beings — even school teachers and superintendents — have not lost their interest in new things. The failure to examine new books critically is often due to the absence of rational standards.

Changing Character of Textbooks

A consideration of the adaptability of textbooks is one of the duties that falls naturally within the province of the supervisor. He looks to them as representing a registration of educational progress. Textbook makers in the main stand for sanity in practice; they are intermediators between tradition and radicalism; they seek to make progress slowly. An investigation of a number of textbooks in arithmetic, ranging over several decades, shows that eliminations and additions have followed fairly closely upon the changes in business life.

The table on the opposite page presents a comparison of the changes that have occurred in a period of sixty-five years in percentage and its applications, as shown by a study of ten textbooks.

It is believed that these texts are fairly typical of the periods in which they were used, although it is not known that they were used more extensively than others.

TABLE XL

Historical Variation of Percentage and its Applications

Date of Publication	Plan: Topical	Plan: Spiral	Plan: Combined	No. Pages	No. Applications	No. Rules	No. Problems	No. Examples	% Book Space	Character of Problems: Oral, Concrete, and Abstract	Character of Problems: Concrete	Character of Problems: Abstract	Loss and Gain	Commission and Brokerage	Stocks and Bonds	Insurance	Taxes	Customs and Duties	Simple Interest	Bank Discount	Partial Payments	Exchange	Equation of Payments	Present Worth	Annual Interest	Promissory Note	Annuities	Partnership	Commercial Discount	Compound Interest
1. 1848	X			15	15	15	267	19	15.0	0	267	0	5	32	10	15	4	12	82						7			13		7
2. 1856	X			112	15	35	648	46	31.3	0	596	52	56	47	15	23	16	21	103	29	11	51	7				43			53
3. 1892	X			81	16	2	856	35	19.0	151	549	56	47	19	44	16	11	11	42	18	13	46			12	11		25	17	27
4. 1893	X			68	15	9	484	30	21.0	86	398	0	48	25	20	8	10	7	26	20	9	34	11	18	5				13	
5. 1895			X	65	13	0	1673	0	24.0	980	618	75	64	18	35	24	17	20	142	40	11	22		21	6					7
6. 1897	X			46	14	4	331	16	13.3	161	170	0	56	18		9	4	6	94	19	10	17		17	10				7	7
7. 1899		X		56	8	0	599	14	12.0	288	191	180			13	5	5		52							11			5	
8. 1904			X	102	15	4	1134	31	26.0	232	800	72	48	31	9	22	45		86	45	7	39						10	14	3
9. 1910			X	50	12	7	610	33	24.0	124	486	0			33	28	18	13	112	22	8	32						15	23	19
10. 1912			X	91	13	0	1144	21	38.0	278	755	0	94	27		32	27	13	115	15	5					4			182	6

It is a matter of regret that when this study was being made we were unable to secure a book that was in current use between 1856 and 1892. It would probably not have revealed any very striking changes, as the 1856 text was the most widely used book during this entire period.

THE FAILURE OF THE SPIRAL PLAN

A significant modification in the general plan of text-book making is indicated by this table as taking place near the close of the last century when Hall's arithmetics, No. 9, afterwards known as the Werner arithmetics, came into vogue. The distinguishing feature of these books was the spiral plan by which each topic was repeated and expanded with mechanical regularity upon a later page. This concept was almost literally epoch making in the field of school method; it practically revolutionized a number of old texts and called forth a flood of new ones. But in their ardor to attain a very desirable end, many authors overworked the device, and, as a consequence, a reaction set in almost immediately. The great virtue of the spiral plan, however, was that it made us more self-conscious with reference to the possibilities of teaching technique. This increased self-consciousness of the value of method has resulted in numerous recent attempts to find a happy combination of the two methods, the spiral and the topical; for it is generally recognized that either alone

is unjustifiable, — a fact which we have discussed in another connection.

The Integration of the Separate Units

There is another very interesting feature of these books, — a feature not readily reducible to tabular form, — that distinguishes the old from the new books. Formerly the different topics or divisions of the subject were in no way connected; each new division of the subject was begun and treated as if it had no connection with anything that preceded or followed. Now these topics or divisions are as closely connected as human ingenuity can make them. Where there were once pages of explanations and definitions relating to the new subject, there are now problems and questions which are intended to bridge the gap. After a half page of explanation, including the "General Rule" and an example, Ray's first problem in Profit and Loss is "If my rate of gain is 25 %, how much should I mark goods for sale that cost me \$8; \$7.50; \$6.25; \$4.75; \$3.87½; \$2.62½; \$1.93¾; .62½¢; 15¢ a yard?" In dealing with this topic the first words of the eighth book listed in Table XL are "What is \$2.50 with 10 % added? with 20 % added?" Seven such questions as these, becoming progressively more difficult, are presented before the suggestion is offered. "The per cent of profit or loss is always reckoned on the cost of the goods." Then an example (an analyzed problem) is presented.

CONCRETE PROBLEMS OF DIFFERENT PERIODS: ILLUSTRATIONS

There is still another feature not presented in the foregoing table, that is entitled to some consideration. We have reference to the changing character of the problems, included in the texts. Beginning with Ray's arithmetic, we have selected three of the very first supposedly concrete problems from each of the texts.

No. 2: — A man owing $\frac{3}{4}$ of a ship, sold 40 per cent of his share. What part of the ship did he sell and what part did he still own?

Out of a cask containing 47 gal. 2 qt. 1 pt. leaked $6\frac{2}{3}$ per cent. How much was that?

A found $5 which was $13\frac{1}{3}$ per cent of what he had before. How much had he then?

No. 4: — A cistern with a capacity of 84 gal. is $41\frac{2}{3}$ per cent full. How many gallons does it contain?

A owed B a certain sum of money. After paying him 20 per cent of the debt, 25 per cent of the remainder, 50 per cent of what then remained, and $83\frac{1}{3}$ per cent of the third remainder, what part of the debt was still unpaid?

In a school of 42 pupils, 7 were in one class, 14 in another, 6 in a third, 12 in a fourth, and the remainder in a fifth. Give the per cent of the school in each class.

No. 5: — A farmer having 400 sheep loses 20 per cent of them and sells 25 per cent of the remainder. How many does he sell?

Corn shrinks 20 per cent from the time it is first husked. How many bushels will 6800 lb. of corn measure after shrinking, allowing 56 lb. to the bushel?

If a certain cloth shrinks $4\frac{1}{3}$ per cent of its length, what is the shrinkage of a piece containing 38 yd. before shrinkage?

No. 8: — If a man sells 12 per cent of his hens and has 66 left, how many had he at first?

If you have read all but 40 per cent of a book, and have read 234 pages, how many pages has the book?

A stenographer can write 95 words a minute. Three months ago she wrote 20 per cent less than now. What was her rate then?

No. 10: — The weight of a live chicken is $4\frac{1}{4}$ lb. When dressed it weighs only 3 lb. What per cent of the live weight is waste?

In his examination in arithmetic a boy had 10 problems out of 12 right. His grade was what per cent?

Emery paper costs $4.63 per ream, emery cloth costs $13.25 per ream. What is the difference in the cost? What is the per cent of difference?

Constants and Variables in Percentage

A cursory examination of these problems shows that the character of problem making has been undergoing changes. One feels that when some of the older authors wrote their problems down, they must have said, "Now see if you can get that." Problem makers now give less attention to the hypothetical disciplinary value of problems and more to their social utility.

Although there has been no tendency whatever to increase the absolute space given to this phase of arithmetic, the ninth column shows that the actual per cent of total book space given to percentage and its applications has actually increased from fifteen to thirty-eight.

As the years have gone by there has been a slight decrease in the number of applications treated. The

number of rules stated dropped to the minimum about fifty years ago. This may be a startling revelation to some of those "modern" schoolmasters who have recently discovered the futility of teaching arithmetic by rules. Practically no rules are presented in any of the new books, and practically none have been presented in half a century. The number of problems, however, has shown a tendency to increase; the number of examples has varied irregularly, but the necessity of using them seems to be about as pronounced as ever. Oral and concrete problems seem to be increasing, while abstract problems (if they may be so named) have practically been eliminated.

Taxes, insurance, and simple interest are the only topics that have a place in each of the ten books.

Gain and loss, commission and brokerage, stocks and bonds, duties, bank discount,. and compound interest and partial payments each appear in eight of the books. Commercial discount appears in seven books, and is clearly increasing in importance. Of these eleven topics, partial payments and compound interest are losing in importance; the others may be said to represent the constants of this part of the curriculum. Apparently there is a persistent demand to retain percentage and its applications because of its interpretative rather than because of its utilitarian value, for most of these topics are not taught for the purpose of making children adept

in certain skills. A utilitarian claim may be established for interest, but such a claim would hardly be considered sufficient to justify the presence of the other subjects.

Perhaps the query may be made as to how such material as this helps a superintendent. It must be admitted that this particular study does not supply him with definite quantitative standards for the judging of textbooks. Perhaps no such standards are possible, and, certainly with reference to some aspects of arithmetic, they are not desirable. But this study does furnish a number of suggestions that should receive consideration when textbooks in arithmetic are being examined. In the first place it indicates what the variables and constants are in percentage. In the second place it supplies a rough basis, as indicated by the number of problems, for estimating the relative emphasis each topic should receive.

The Shifting Content of Mensuration[1]

For the purpose of studying some topic intensively a second investigation was made, — this time with mensuration. Only two things were considered: whether there had been any change (1) in content or (2) in the relative emphasis of the topics as indicated by the amount of space devoted to them. Thirty-one

[1] The authors are indebted to Mr. L. O. Bright for most of the material in the investigation of this topic.

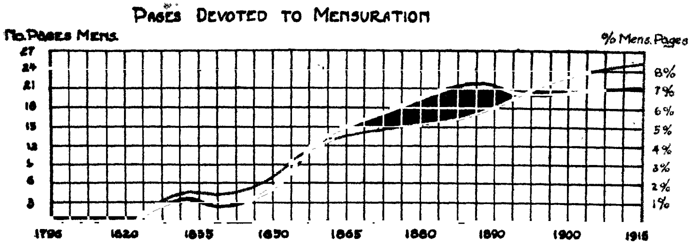

Fig. I.

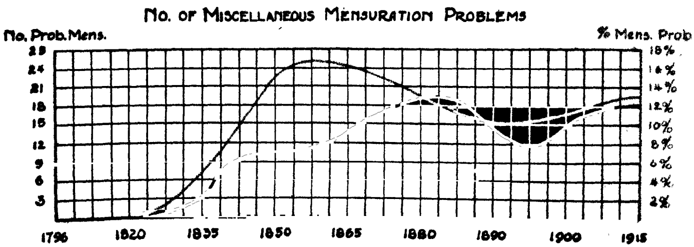

Fig. II.

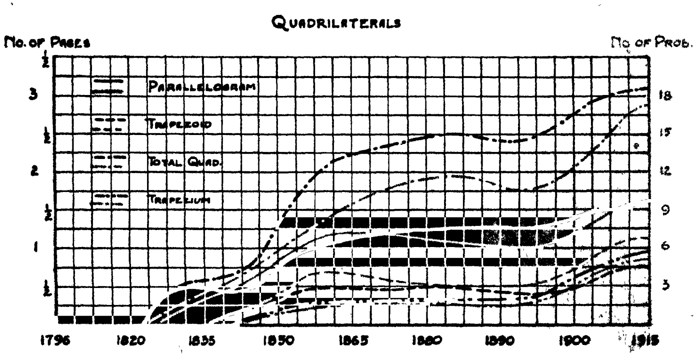

Fig. III.

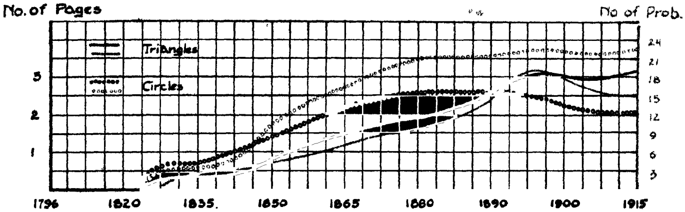

Fig. IV.

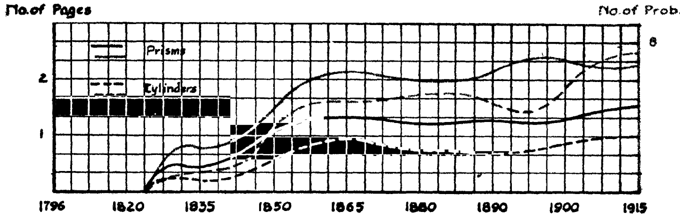

Fig. V.

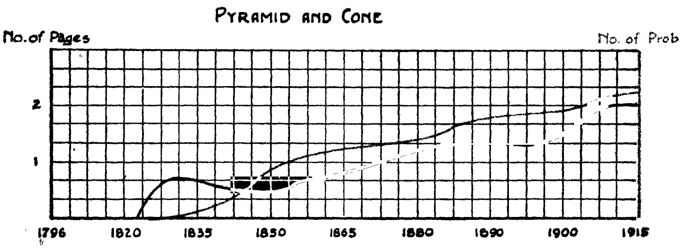

Fig. VI.

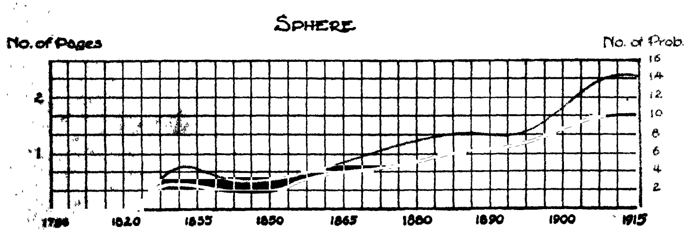

Fig. VII.

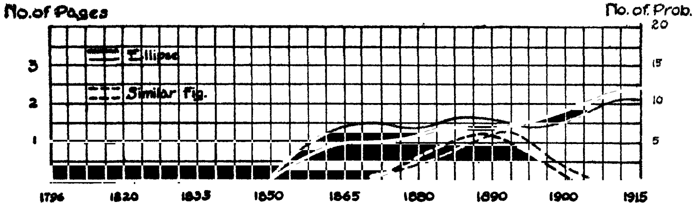

FIG. VIII.

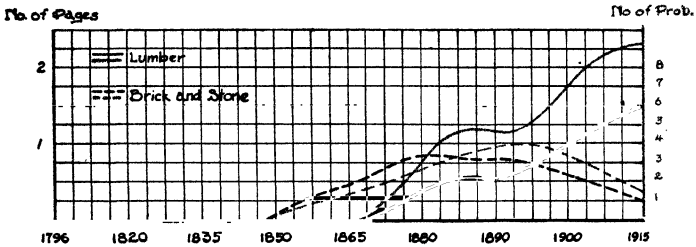

FIG. IX.

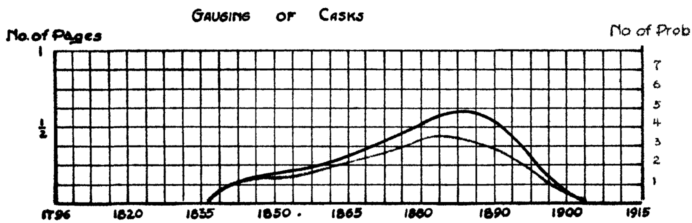

FIG. X.

grammar school texts, ranging according to their dates of publication, from 1796 to 1912 were examined. The amount of space and the number of problems appearing under each of the following topics were checked: Quadrilaterals, including trapezoids, trapeziums, and parallelo-

grams; circles; triangles; prisms; cylinders; pyramids and cones; spheres, ellipses; similar figures; lumber measure; brick and stone measure, and gauging of casks.

The results can be shown more clearly by the graphs here given than by the original table. In each instance the arithmetic mean of the period is taken as the ordinate of the period. In Figure I the black line shows the number of pages devoted to mensuration, indicated by the scale at the left. The red line shows the percentage of the total number of pages devoted to mensuration. From this figure one can see that mensuration came into use in textbooks shortly after 1820 and that it has gradually increased in importance until in 1912 about twenty-one pages or 8 per cent of the whole book was given to it.

Figure II shows the rather irregular increase in the number of miscellaneous mensuration problems appearing in textbooks. In Figures III to X the black lines denote the number of pages, read from the scale to the left, and the red lines show the number of problems given to each topic.

Quadrilaterals have gradually risen in importance from about one third of a page with two problems to three pages with eighteen problems. The trapezium, which appeared in the texts about 1845, about ten years after the trapezoid, is receiving less attention than the

trapezoid. The parallelogram, which appears in the texts about 1825, now receives almost twice the space of the trapezium or the trapezoid. From the earliest introduction of mensuration, we have had circles, triangles, cylinders, prisms, pyramids and cones, and spheres, and each of these has gradually increased in importance. The ellipse, which came in about 1880, disappeared about 1900. The study of similar figures has slowly increased since 1850. Brick and stone measure have disappeared, but lumber measure has increased since 1870. Gauging of casks lasted from 1840 to 1900.

A Comparison of Five Elementary School Arithmetics

A critical study of five elementary arithmetics now in use was made for the purpose of comparing them as to the similarity of their structure. It was presumed that uniformities would be readily discoverable and that they would furnish the uninitiated with standards for estimating the structural character of any elementary text in arithmetic. A summary of this analysis is presented in Table XLI, the meaning of which becomes clear when read thus: Textbook number I contained 273 pages, 3 chapters, 11 topics, 37 lines of footnotes, 228 explanations, 183 pictures, 1949 examples, 1526 written exercises, 905 review problems, 54 definitions, etc.

TABLE XLI

TABLE SHOWING TOTALS FOR VARIOUS ITEMS IN FIVE ELEMENTARY ARITHMETICS

	I	II	III	IV	V
Number of pages	273	256	243	264	135
Number of chapters	3	3	4	5	
Number of topics	11	12	12	12	8
Number of notes (lines)	37	0	205	18	153
Number of explanations	228	73	163	108	37
Number of pictures	183	162	29	62	51
Number of examples	1949	1201	335	1214	605
Number of written exercises . .	1526	849	830	1026	1021
Number of review problems . .	905	148	395	730	124
Number of definitions	54	21	40	48	18
Number of rules	15	5	52	15	
Number of drills	32	46	30	49	39
Number of home problems . . .	84	84	143	124	200
Number of school problems . . .	39	29	26	95	68
Number of farm problems . . .	101	176	193	123	37
Number of business problems . .	220	213	338	336	232
Number of arithmetical problems .	481	494	282	263	444

It will be seen from the above lists that dissimilarity rather than similarity is the custom. Note, for example, that the number of notes varies from none to 205, the number of lines of explanations ranges from 37 to 228, the number of pictures from 29 to 183, the number of examples from 335 to 1949, the number of written exercises from 526 to 1026, the review problems from 124 to 905, the number of rules from none to 52, and so on. That part of the table which deals with the more strictly arithmetical relationships shows that writers of

arithmetics are not agreed as to the standards that should prevail in the construction of textbooks. Less diversity is manifest in those topics that deal with the applications of the arithmetical concepts and principles.

Greater similarity is found when we examine the order in which the topics are treated in these books. The next table presents this information.

TABLE XLII

ORDER OF TOPICS IN FIVE ELEMENTARY ARITHMETICS

I	II	III	IV	V
1. Counting	1. Counting	1. Counting	1. Lines, Angles	1. Counting
2. Addition	2. Addition	2. Addition	2. Counting	2. Size and position
3. Subtraction	3. Subtraction	3. Subtraction	3. Addition	3. Combination Addition and Subtraction
4. Multiplication	4. Multiplication	4. Multiplication	4. Subtraction	4. Multiplication
5. Division	5. Division	5. Measuring	5. Multiplication	5. Division
6. Measures	6. Roman Numerals	6. Division	6. Division	6. Measurements
7. Roman Notation	7. Fractions	7. Roman Numerals	7. U. S. Money	7. U. S. Money
8. U. S. Money	8. Measurements	8. Fractions	8. Measurements	8. Roman Numerals
9. Fractions	9. Analysis	9. Decimals	9. Fractions	9. Mail
10. Decimals	10. Decimals	10. Bills	10. Ratio	10. Bills
11. Bills	11. Bills and Statements	11. Factors and Multiples	11. Bills	11. Fractions
12.	12. Farm Problems	12. Percentage	12. Percentage	12. Long Division
				13. Markets

RELATIVE EMPHASIS OF GIVEN TOPICS

It will be seen that about the same topics are treated and that they are treated in about the same order in all of the books. Clearly counting, addition, subtrac-

tion, multiplication, division, measurement, and fractions are the constants; such topics as ratio, Roman numerals, analysis, and percentage are variables.

The next table gives some notion of the relative emphasis these topics are receiving in each of the five arithmetics.

It seems to us that the differences are more interesting than any central tendencies that might be computed, for these, after all, would really not be descriptive of practice. Considering the fact that the first two books on the list are enjoying a wide sale and the others are having almost no sale at all, we were disposed to examine the first two more closely in the hope that similarities may be noticeable. But such an examination was futile; it shed almost no light upon the relative values of these books.

A Dictionary of Terms and Names

As a final resort, therefore, we had every word in the first fifty pages of these books counted and its frequency noted. This may appear to be a foolish task. But we were really interested in determining whether these books are arithmetics or something else. It seems logical to assume that an arithmetic should fix certain arithmetical terms, principles, or concepts. Recurrence or repetition is necessary to insure this fixation. If this repetition recurs only very

TABLE XLIII

TABLE SHOWING NUMBER OF ITEMS IN ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION IN FIVE ELEMENTARY ARITHMETICS

	ADDITION					SUBTRACTION					MULTIPLICATION					DIVISION				
	I	II	III	IV	V	I	II	III	IV	V	I	II	III	IV	V	I	II	III	IV	V
Lines of notes to teachers	10	0	7	0	53	3	0	53	0	11	11	0	29	0	22	0	0	52	0	15
Number of explanations	14	4	10	6	2	12	7	10	7	3	18	8	27	18	3	33	13	31	17	3
Number of pictures	50	14	0	4	9	13	18	0	4	0	5	20	0	5	7	21	29	0	9	8
Number of examples	207	163	93	207	123	127	200	53	172	95	364	134	12	154	91	822	304	52	242	267
Number of mental exercises	198	86	54	60	138	100	121	38	65	145	222	167	60	164	166	251	47	60	321	214
Number of review problems	114	5	13	114	23	133	10	18	158	19	317	38	57	139	23	65	37	15	222	11
Number of definitions	2	1	2	2	1	4	2	2	2	3	4	3	5	1	4	14	3	5	9	4
Number of rules	0	0	3	1	0	0	0	3	1	0	2	0	6	4	0	4	0	5	0	0
Number of drills	2	8	1	11	10	0	8	0	1	2	111	16	9	16	11	0	8	2	15	12

TABLE SHOWING NUMBER OF ITEMS IN NUMBERS, MEASURES, FRACTIONS, AND DECIMALS IN FIVE ELEMENTARY ARITHMETICS

	NUMBERS					MEASURES					FRACTIONS					DECIMALS				
	I	II	III	IV	V	I	II	III	IV	V	I	II	III	IV	V	I	II	III	IV	V
Lines of notes to teachers	8	0	29	0	15	5	0	20	0	11	0	0	15	0	0	0	0	0	0	0
Number of explanations	35	11	3	9	8	10	4	36	17	11	90	16	19	9	5	10	10	9	0	0
Number of pictures	25	53	0	12	8	46	10	25	12	14	19	16	1	0	0	0	0	0	0	0
Number of examples	41	47	21	55	4	26	36	0	0	0	279	259	40	203	4	73	58	49	0	0
Number of exercises	158	172	35	167	105	351	137	260	114	162	96	95	176	56	11	93	18	30	0	0
Number of review problems	106	2	0	0	8	58	12	122	6	25	98	35	75	29	0	18	0	23	0	0
Number of definitions	2	2	2	8	3	12	0	6	14	0	11	10	10	2	2	2	1	1	0	0
Number of rules	0	0	0	1	0	0	0	16	0	0	7	0	9	9	0	2	7	7	0	0
Number of drills	1	0	0	0	0	13	6	16	6	2	0	0	2	0	0	2	0	0	0	0

infrequently, the student, not being familiar with the laws of habit formation, will drift aimlessly through the subject, spending two or three years acquiring a few fundamental skills that should be assured him at the outset.

It was such fundamental propositions as these that induced the authors to prepare Table XLIII. An examination of the Table gives one certain common sense notions as to why some of the books are necessary and others failing. It will be observed, for example, that Book V gives little attention to explanations, to review problems, or to common and decimal fractions. While Book IV emphasizes review problems in the fundamentals, it does not do so consistently in the other operations, and it neglects to give any considerable attention to fractions. Book I is the most consistent in the emphasis it gives to the various topics.

In the first fifty pages of these books 94 proper names are used 342 times. The names appearing most frequently are Ella and Kate, which appear 6 times each, Helen 8, Henry 9, Carl 12, Fred and James 16 each, Frank 24, Mary 27, and John 40. There are in these same pages 224 words beginning with *c*, and these words are used a total of 1403 times. As a sample of the variety of things that may appear under a single letter of the alphabet we print two of these tables, — the tables listing the words beginning with *s* and *w*.

TABLE XLIV

Words Beginning with "S" with Their Frequencies in Five Elementary Arithmetics

Words	I	II	III	IV	V	Words	I	II	III	IV	V
sacks					5	shad	1				
saddle					2	shall			1		5
sale					1	share					3
Sam	1				7	she	7	11	1	31	28
same		1	10	2	2	sheep	6	10	11	1	3
Sara	1			3		sheet				3	
satisfactory			1			shelf	1		1	28	
Saturday			1		2	shock	1		1		8
sat				1	1	short		1	9	3	2
sauces					1	should	4		26		11
save	1		3	2	3	show		7	24	3	6
savings	1					shot	1				
saw			1		2	shrubs	1				
say	5		1		3	side	1	2	4	9	4
school	3	2	2	6	21	sidewalk				1	
scissors					4	sight	10	1	2		
score	2				8	sign	6	13	10		
scored					9	signal					2
sealed					1	silk				1	1
search					1	sill					1
season				1		simple			2		1
seat			1	5	5	sing					1
seated	1			1	1	single			2	2	1
second	1	1	2	13	7	sister		8		1	3
sections	6		2		1	sitting				1	
see	2	3	10		8	situation					1
seeds		3				six	22	1	9	5	4
seek					1	sixteen	1		2	1	1
seen			2		1	sixths		6	1	4	
select				1	1	sixty	4		2	4	
sell	4		5	2	6	size			4		2
selling	1		1	1	1	skates				1	
send					1	sketches				2	
sent				1	3	sleeps				2	
separate			3		1	slightly			1		
session				1		slip				3	
set			2		7	slowly					1
seven	20	1	2	3	3	small			2	1	5
seventeen	2		1	1	1	Smith					6
seventy	5			5		so	3	1	10		4
several					3	soap	1			1	1

TABLE XLIV — *Continued*

WORDS BEGINNING WITH "S" WITH THEIR FREQUENCIES IN FIVE ELEMENTARY ARITHMETICS

WORDS	I	II	III	IV	V	WORDS	I	II	III	IV	V
soda					1	stimulate . .					1
sold	7	5	10	5	16	stock . . .	1				
soldiers . . .	1				3	stopped . . .					1
solve				10		store			4	4	6
some			28	2	8	stormy . . .				1	
somethings .			2		1	story . . .	2		6	1	1
sometimes . .	5	1				straight . .			1	7	
soon	1		1		1	strawberries .			1	4	
sounds . . .	1				1	street . . .				1	5
soup					2	strengthen . .			2		
source . . .			1			strings . . .	1			2	
south . . .					3	strip		1		1	2
souvenirs . .					1	stripes . . .	2				
spaces . . .			2	4	2	stroke . . .	7				
spade . . .					2	struck . . .				1	
sparrows . .	2			1		study . . .			4	5	
speak . . .			1			stumble . .			4		
speckled . .				4		subdivisions .			1		
spelling . . .				6		subscribers .					1
spend . . .	2		3	6	2	subtract . .	12	10	12	7	2
spider . . .		2				subtraction .	3	2	17	1	7
splints . . .	3		16			subtrahend .			3		1
spoke . . .					2	succeeding . .					1
spools . . .	1	2				such			2		3
sponge . . .	1					sugar . . .			4		2
sprinkling . .					2	suggest . . .			4		1
spruce . . .				2		sums . . .	19	13	13	1	3
square . . .	34	28	4	16	11	Sunday . . .	3				2
stable . . .	1					supper . . .	1				
stacks . . .				3		supply . . .	4			7	1
stage . . .			1			supposing . .					1
stairway . .		6				surely . . .			1		
stamp . . .	4				27	surface . . .			2		
stand . . .	21		1		6	Susan . . .				2	
standard . .			1		4	swallows . .	1			1	
start	8		2		3	swing . . .			1		
state	2			4	1	syrup . . .					2
stays . . .	3				1	Total . . .	279	182	304	271	381
sticks . . .		65	1	8	6	No. different words . .	62	25	74	67	113
stiff					1						
still					3						

TABLE XLIV — *Continued*

WORDS BEGINNING WITH "W" WITH THEIR FREQUENCIES IN FIVE ELEMENTARY ARITHMETICS

WORDS	I	II	III	IV	V	WORDS	I	II	III	IV	V
wagon	4	5	3	1		whom			2		
walk			1		7	why			1		
wall					1	wide		4	1	2	2
walnut					1	width				1	
wanted	1				1	Wilbur					1
warm					1	will	3	6	11	4	20
was	2		1	1	17	William	2		2	1	2
watch			2			willow	1				
water		7	1	1	3	win				1	1
wave					3	window		1	1	2	2
way		5	14	1	2	winning					
we				27	37	wise			1		
wear					3	wished					5
weaving					1	wishes	1			1	2
Wednesday	1			1	3	with		6	10	14	23
week	14		1	2	19	without			4		3
weigh	2		13			women	2				
well			3			won				1	
went	1		1	2	9	wood		1	1		
were	2	2	7	19	28	woodchucks				2	
what	20	6	27	25	40	words	2	1	2		1
wheel	2	7			3	work		2	31	1	4
when	2		7	1	12	worms				4	
where			2			worth	4		2		12
whether			1			would			3		7
which	1	1	28	6	21	wren	2				
whichever		1				write	43	13	51	31	22
while					3	written	25	8	27	8	1
whistle				1	1	Totals	140	92	298	141	239
white	3	9		5	2	No. different words	23	19	35	29	40
who			3	1	3						
whole		7	6	1							

THE RECURRENCE OF ARITHMETICAL TERMS

The next table, which presents the distribution of the arithmetical terms in each of the arithmetics, shows

TABLE XLV

TABLE SHOWING ARITHMETICAL TERMS OF FIRST FIFTY PAGES

Arithmetical Terms	Frequencies of Arithmetical Terms				
	I	II	III	IV	V
add	20	2	20	17	23
addition . .	2	2	16	1	2
as many as .	0	0	0	0	0
as much as .	1	0	0	0	0
count . . .	32	6	12	8	20
counting . .	0	0	0	0	10
difference . .	10	1	2	0	2
divide . . .	0	32	11	6	7
divided by .	0	28	0	0	0
dividend . .	0	0	7	0	0
division . .	0	0	13	1	3
divisor . . .	0	0	0	5	0
equal . . .	0	9	9	14	8
fraction . .	0	0	3	0	0
how many .	272	328	54	112	262
how much .	15	4	24	35	45
less, less than	81	24	0	0	2
minuend . .	0	0	2	0	1
minus . . .	0	0	2	0	0
more than .	11	0	0	9	14
multiplication	0	0	12	1	2
multiply . .	1	1	7	2	3
plus . . .	0	0	2	0	0
quotient . .	0	0	4	0	0
remainder .	1	0	12	0	0
solve . . .	0	0	0	1	0
subtract . .	9	12	13	5	1
subtraction .	1	0	16	1	8
subtrahend .	0	0	3	0	1
sum . . .	17	0	12	1	2
take away .	0	1	0	19	0
times . . .	3	43	24	21	28
units . . .	5	0	2	0	0
what part of .	0	4	0	4	0
Totals .	481	494	282	263	444

clearly that there has been no conscious attempt on the part of any of these authors to organize their books so as to insure the fixation of certain fundamental arithmetical expressions. We believe this to be a defect common to many arithmetics.

An Analysis of Grammar-grade Texts

Four current grammar-grade textbooks in arithmetic were selected for careful analysis. First of all, each text was inspected with a view to finding the particular topics included. Second, the number of problems on each topic was determined. Third, the number of lines of introductory description was counted. Fourth, the teaching devices were counted. The texts were then compared in order to form an estimate of the degree of agreement which existed among them. It was immediately apparent that there is agreement on the part of the writers as to certain topics which are to be taught. All of these authors present addition, multiplication, division, subtraction, denominate numbers, fractions, percentage, measurement, interest, and the like. Two of the authors, however, would use no topics under the head of banking and exchange. Three had selected no topics under the head of discount. On the other hand, one text had selected topics in the field of insurance, taxes, public expenditures, transportation, and the like. The children in a school system using

one of these textbooks would have access to material touching these lines, while children in schools using the other texts would be denied the opportunity of coming into contact with such topics, unless the teachers or supervisor consciously introduced them. It would be worth while for each supervisor to make some such analysis as this of the textbook that is now being used, in order to find whether or not topics of questionable value are being presented, and whether important topics are being omitted.

A further analysis was made of these books in order to determine the relative emphasis given to the different topics. For example, it was found that the book placing the most emphasis upon addition presented five times as many problems as the book giving the least emphasis to addition. The supervisor should ask himself the question: Do the children under instruction in this particular instance need the material in the proportion indicated by this analysis? Are there too many problems, or are there not enough problems? Again, the same variation was present in the case in fractions. The book placing the most emphasis upon fractions had almost five times as many problems as the book giving the least emphasis to fractions. What is the cause of this variation? Who is right? Do we need hundreds of problems in fractions, or do we need a few problems only? The supervisor or teacher should know whether

or not the textbook in use provides enough problems for her particular purpose. Certainly it is an unsafe proposition to rely merely upon the textbook makers in this particular.

Again, certain texts make a great point of review exercises. One text has sixteen times as many review problems as another. To what is this difference due? The supervisor or teacher should know the extent to which review exercises are profitable for the class under instruction. With this variation in emphasis on the textbooks, it is clearly unwise to rely solely upon a textbook.

The Only Adequate Standards are Common-sense Standards

The supervisor who is intent upon results in arithmetic needs a more secure basis than the foregoing for the judging of textbooks. Adequate scientific standards for judging textbooks have not been determined. It is clear that textbooks should be adopted on the basis of what is in them, on the basis of the selection of topics, the sequence of topics, and the gradation of the material, but fixed and unalterable definitions cannot be determined for these items. They will continue to be modified in light of shifting social needs.

It is probable that greater agreement would have been found had more books been studied and compared. The books studied were chosen because they were be-

lieved to be representative. They at least warrant the assumption that books must continue to be selected upon the basis of common-sense standards, and these standards usually relate to whether the books are closely organized and teachable. Naturally the application of such standards may permit the incorporation of much obsolete material in the books. This material can be eliminated only when other criteria are used for judging it. Reference must be made to social conditions to determine whether the material in the book is still serviceable and reference must be made to the personal needs of children to determine whether the material may be learned most economically. Definite criteria of this sort are not available.

CHAPTER X

ALGEBRA AND GEOMETRY IN THE GRADES

The Recommendations of the Committee of Ten

The Committee of Ten, which reported in 1893, recommended instruction in algebraic symbols and in simple equations. It is also stated "that a child's geometrical education should begin as early as possible. — At the age of ten years for the average child systematic instruction in concrete or experimental geometry should begin and should occupy one hour per week for at least three years." It was not presumed in this report that algebra and geometry should appear as separate subjects, but that they should be taught in connection with arithmetic. The recommendations followed closely the prevailing practice in Europe. Although considerable opposition has been voiced from time to time to this part of the report of the Committee of Ten, many mathematicians and superintendents now indorse the early introduction and use of algebraic formulæ and certain fundamental concepts of geometry.

The Attitude of Superintendents with Reference to the Use of Algebraic Symbols

The authors, in a recent investigation, sought information from a large number of superintendents in regard to this point.

A little less than half of the superintendents in cities of 4000 and over, and more than one hundred county superintendents, replied to the queries as to whether algebraic symbols should be taught in the grades, and if so, in what grades. Their replies appear in the tables.

Opinions were expressed by seven hundred ninety-five superintendents; five hundred thirty-nine of whom expressed themselves definitely in favor of teaching Algebra symbols in the grade. Two hundred forty-one expressed themselves definitely as being opposed to the teaching of algebraic symbols. Fifteen expressed themselves doubtfully in regard to the issue.

TABLE XLVI

Geographical Section	Yes	No	Questionable	All	II	III	IV	V	VI	VII	VIII
North Central	182	97	11	290	6	7	12	17	32	103	182
North Atlantic	167	68	3	238	4	6	9	15	37	83	167
Western . .	33	20	0	53	0	0	1	1	7	16	53
South Central	57	17	0	74	4	5	6	8	21	50	57
South Atlantic	27	3	0	30	1	1	1	5	10	17	22
Counties . .	73	36	1	110	5	6	8	10	20	44	66
	539	241	15	795	20	25	37	56	127	318	527

TABLE XLVII

(Reduced to per cents)

	Yes	No	Question-able	All	II	III	IV	V	VI	VII	VIII
North Central . .	62.7	33.4	3.9	100	3.3	3.8	6.6	9.3	17.6	56.6	100.0
North Atlantic . .	70.2	28.5	1.3	100	2.3	3.6	5.4	8.9	22.1	52.7	100.0
Western	62.3	37.7	0.0	100	0.0	0.0	3.0	3.0	21.2	48.5	100.0
South Central . .	77.0	23.0	0.0	100	7.0	6.3	10.5	14.0	36.8	87.7	100.0
South Atlantic . .	90.0	10.0	0.0	100	3.7	3.7	3.7	18.5	37.0	62.9	81.5
Counties	66.4	32.7	.9	100	6.9	8.2	10.9	13.7	27.4	60.3	90.4
	67.8	30.3	1.9	100	3.7	4.6	7.0	10.4	23.5	59.0	97.8

To the left of the dotted line, Table XLVI reads 182 superintendents out of 290 in the North Central states favor instruction in algebra in the grades, 97 are opposed to it, 11 consider it questionable. The figures to the right of the dotted line show in what grades these superintendents think algebra should be taught. It will be observed that the totals to the right of the dotted line do not agree with the totals to the left of the dotted line. This is due to the fact that a single superintendent may have indicated that the subject should appear in more than one grade, and his reply is consequently recorded for each of the grades indicated. It would have been possible for each grade in the North Central states to have received 182 votes, and so on for each of the geographical sections.

Two thirds of the superintendents advocate the teach-

ing of algebra in some of the grades. Almost one third are opposed to instruction in the subject.

These superintendents testified only with reference to the use of algebraic symbols. These tables supply no information whatever as to the extent to which definite operations like simple equations, simultaneous equations, quadratic equations, and the like, are advised. Although a small but insignificant number of superintendents advocate the use of the symbols in the second, third, fourth, fifth, and sixth grades, it is not until the seventh and eighth grades are reached that any considerable number urge such instruction.

From Table XLVII, which presents the facts of Table XLVI distributed in per cents, it will be seen that more than one half of those favoring the introduction of the symbols of algebra in the grades maintain that they should appear in the seventh grade, and the vote of this group is almost unanimous in favor of their use in the eighth grade.

Attention is directed to the fact that the South Central and South Atlantic states show a slightly larger percentage of superintendents who favor the use of algebraic symbols than do the other sections of the country. This is of especial interest, as these two groups of administrators urge more strongly than the other groups the use of the symbols in the lower grades. A large percentage of the South Central superintendents mention the sec-

ond, third, and fourth grades, and a large percentage of the South Atlantic superintendents mention the fifth and sixth grades as the grades in which the symbols should appear.

Variation in Attitude of Superintendents in Cities of Different Sizes

The query arises as to whether or not the attitude of the superintendents in the large cities is the same as that of the superintendents of the small cities.

These data consequently were distributed again in order to reveal differences, if any, in the attitude of the superintendents of schools in cities of different size.

TABLE XLVIII

Use of Algebraic Symbols

Size of City	Yes	No	Questionable	All	In What Grade?						
					II	III	IV	V	VI	VII	VIII
1,000,000 and over	2	0	0	2	0	0	0	1	0	1	1
200,000 to 999,999	11	5	0	16	0	0	0	0	2	7	10
100,000 to 199,999	8	3	0	11	1	1	0	2	2	4	8
50,000 to 99,999	18	12	0	30	0	0	0	1	2	7	18
30,000 to 49,999	29	13	0	42	0	0	1	1	3	15	29
20,000 to 29,999	33	12	0	45	2	5	5	5	13	22	32
15,000 to 19,999	29	14	3	46	1	1	2	2	9	20	29
10,000 to 14,999	56	36	3	95	0	0	2	3	11	36	56
8,000 to 9,999	55	26	2	83	3	4	8	9	21	35	55
4,000 to 7,999	225	84	6	315	8	8	11	22	44	127	223
	466	205	14	685	15	19	29	46	107	274	461

TABLE XLIX

(Above table reduced to per cents)

Size of City	Yes	No	Question-able	All	In What Grade?						
					II	III	IV	V	VI	VII	VIII
1,000,000 and over	100	0.0	0.0	100	0.0	0.0	0.0	50.0	0.0	50.0	50
200,000 to 999,999	68.8	31.2	0.0	100	0.0	0.0	0.0	0.0	18.2	63.6	91
100,000 to 199,999	72.7	27.3	0.0	100	12.5	12.5	0.0	25.0	25.0	50.0	100
50,000 to 99,999	60	40.0	0.0	100	0.0	0.0	0 0	5.5	11.0	38.8	100
30,000 to 49,999	69	31.0	0.0	100	0.0	0.0	3.5	3.5	10.3	51.6	100
20,000 to 29,999	73.3	26.7	0.0	100	6.1	15.1	15.1	15.1	39.4	66.7	100
15,000 to 19,999	63	30.4	6.6	100	3.4	3.4	6.8	6.8	31.0	69.0	100
10,000 to 14,999	58.9	37.9	3.2	100	0.0	0.0	3.6	5.4	19.8	64.3	100
8,000 to 9,999	66.22	31.6	2.5	100	5.5	7.3	14.5	16.4	38.2	63.6	100
4,000 to 7,999	71.4	26.0	2.0	100	3.5	3.5	4.9	9.8	19.6	56.4	100
	68.	30	2.0	100	3.2	4.1	6.3	9.8	22.8	58.8	99

No new facts are revealed for cities of any size by distributing the replies on the basis of the grades in which the symbols are taught. In the form, however, in which the material is presented in this part of the table it will be easy for the reader to be misled. The totals for the grades in Table XLVIII are swelled because some superintendents specified more than one grade. Again the per cents in Table XLIX are based upon the grade distribution of Table XLVIII and not upon the 685 superintendents who replied to the questionnaires.

While very little of scientific value is known about the actual advantage to be gained from the use of algebraic symbols in arithmetic, the working supervisor will be inclined to attach significance to the opinion of the

majority of the superintendents throughout the country. In the absence of better data, there is some reason for this sort of response.

Mr. Van Houten discovered that only three school superintendents out of 148 required the omission of algebra from the course of study, and that a considerable number of them offer one full term of it above the sixth grade.

Algebraic Topics Taught in the Grades

Superintendents in a number of typical American cities were asked to specify what topics should appear in a graded course in algebra. Their replies are listed in the following tables:

TABLE L

Topics Taught in Elementary Grade Algebra

Topics	Cities	Per Cent of Cities Teaching Algebra
Equation with one unknown	95	98.9
Equation with two unknowns	36	37.5
Addition and Subtraction	53	55.2
Multiplication and Division	44	46.0
Factoring	35	36.0
Fractions	49	51.0
Quadratics	7	7.3
Schools using a separate text algebra	16	16.6

This table means that 95 of the cities replying give instruction in equations with one unknown and 36 in

equations with two unknowns, 53 in addition and subtraction, 49 in fractions, 7 in quadratics. It is also shown that 98.9 % of all the cities teaching algebra provide instruction in equations with one unknown, 37.5 % in equations with two unknown, 55.2 % in addition and subtraction, and so on. Sixteen of the schools represented, or almost 17 %, use separate texts in algebra.

Views of Present-day Writers

Recent writers on the subject of teaching of arithmetic, such as Young, Smith, Brown and Coffman, are inclined to favor the use of algebraic symbols. The latter writers say: "In the latter part of the sixth and in the seventh and eighth grades the teacher should not hesitate to introduce letters to represent numbers. It is no more unnatural, after a short time, for the pupil to use 'C' to represent the cost, 'I' to represent interest, 'V' to represent volume, and 'N' to represent a number than for him to use N. Y. to represent New York or A.M. to represent the time before noon. The pupil should recognize the symbols of algebra as a shorthand method of indicating magnitudes. Arithmetic does not become algebra by the use of letters instead of numbers. The solution of many of the problems of common and decimal fractions, or ratio and proportion, percentage and mensuration, is greatly simplified and abridged by the use of letters for the magnitudes which they represent."

TABLE LI

GRADE OCCURRENCES OF GEOMETRY

Grade	Cities	Per Cent of 149
7 B	1	0.6
7 A	2	1.3
8 B	4	2.7
8 A	19	12.0
Schools offering one semester only	18	12.0
Total schools offering Geometry	22	14.8
Schools offering Geometry and not Algebra	2	

TABLE LII

TOPICS IN GEOMETRY TAUGHT

Topics	Cities	Percentage of Schools Teaching Geometry
Constructive Geometry	21	95.4
Similar figures	11	50.0
Schools using text in Geometry	2	10.0

An examination of the second of these tables leads one to suspect that the geometry used is very elementary in character. Only two schools use a textbook in this field.

While it can be safely asserted that there is a marked tendency to give instruction in the simpler forms of algebra in the two upper grades, no such statement can be made with reference to geometry.

CHAPTER XI

PROBLEMS RELATED TO CURRENT BUSINESS LIFE

The Articulation of Subjects with Life Reduces Waste

In every field of human activity efforts are being made to economize time. Usually these efforts result in a standardization of the processes employed in carrying on the work. Frequently economy of effort is secured by the introduction of time and labor saving devices. School officers everywhere are sympathetic with the movement to secure more satisfactory results in less time, but they are not always conscious of the agencies by which this may be accomplished. Certainly one means at the disposal of every superintendent for improving the efficiency of his schools is that of reconstructing the internal organization of the schools. Heretofore superintendents have devoted much of their time and energy to this problem, because it was close at hand and they could readily see the beneficial results that they were securing by the reorganizations they adopted. Im-

portant as details of management and of organization are, they are entitled to no more consideration and are of no more importance in removing unnecessary waste and friction than are the adjustments of the school to shifting social and industrial conditions. Just to the extent that a proper articulation can be made between the materials of the school and those social situations which they represent, waste may be avoided.

The Evolution of Subjects of Study

It is a fact that every subject of study had a long period of preconscious evolution before it was consciously formulated. During this time the race was discovering that some of its experiences had a similarity and that these experiences were of service in securing adjustments of a like character. Obviously some of these experiences were transient in character and were soon forgotten. Others occurred so frequently and with such intensity and touched the lives of so many people that they were considered as vital and necessary to the lives of all the people. These experiences were saved, and many of them were eventually incorporated in the various subjects of study. Each subject of study, therefore, represents a special attempt at environmental adjustment. Each subject of study should give one control over some special phase of his environment. Now the unfortunate thing is that many teachers forget the con-

crete situations that gave rise to the experiences that are incorporated in any particular study and teach the subject as if it were an end in itself instead of a means to an end. Since each subject is merely a series of related problems, it should be taught not only to show its integrity, but so that its facts and processes may function in situations outside the school that are similar to those that gave rise to the materials included within the subject.

There is no subject that has suffered more in this particular than arithmetic. Teachers have been clinging to obsolete phases of the subject, *i.e.*, to aspects that are no longer used in the business world, with the hope that they might still be socially serviceable. Some teachers, however, have justified the retention of certain divisions of the subject on disciplinary grounds; for example, the Euclidean method of finding the greatest common divisor. It is doubtful whether any one should argue for the retention of any subject in the curriculum purely on the ground of its mind-training value, and yet every one desires that the mind be trained. Surely the mind will get as much training from exercise upon materials that are useful as from exercise upon materials that are no longer useful. It is for this very reason that textbook makers in the field of arithmetic and superintendents are eliminating much of the material that the business world has ceased to use. Such topics as the

greatest common divisor and the least common multiple (when numbers are not readily factorable), obsolete tables in denominate numbers, troy weight, apothecary weight, circulating decimals, cube root, progressions, compound proportion, and the like have disappeared or are disappearing from our textbooks and courses in arithmetic.

Dangers Inherent in the Localization of Subject Matter

Believing that modern social theory had affected the opinions of school superintendents everywhere with reference to whether time could be saved by relating the problems in arithmetic more closely to industries, occupations, and the like, a series of questions were submitted to them bearing upon this point. We realize that there are certain dangers in attempting to make such adjustments. No school curriculum should be completely localized, for the reason that many of those trained in any particular community will live elsewhere. Moreover, localized school curriculums would accentuate differences between communities. It seems to us that the primary purpose of the elementary school is to increase the homogeneity of our general population. On the other hand, the children should see that the material included in arithmetic is of service in carrying on the work of the community.

The Value of the Socialization of Arithmetic

There are some phases of arithmetic that are needed in practically all industries and occupations in every part of the country. These are the things which should receive the greatest emphasis. Seven hundred and sixty-two superintendents furnished testimony as to whether or not time can be saved by securing a closer correlation between the subject matter of arithmetic and the situations in which it is supposed to function. The first two tables show how these answers were distributed. It will be noted that 551 or 72.3 % of those replying were of the opinion that waste could be reduced by such an arrangement. As a matter of fact this total should be increased by 9.3 %, as 71 additional individuals, although a little uncertain, really belong in the affirmative column. It will be noted that about 13 % were of the opinion that nothing could be gained by such a procedure. The greatest conservatism was shown by the superintendents of the North Central and the North Atlantic states, — where less than 70 % maintained that time may be saved in this way; but the difference between these geographical areas and the other sections of the United States is not so great as it seems, for it will be noted that 10.4 % in the North Central and 9.7 % in the North Atlantic states *thought* that time might be saved by increasing the reality and concreteness of the problems.

TABLE LIII

CAN TIME BE SAVED BY RELATING PROBLEMS TO INDUSTRIES, OCCUPATIONS, ETC., OF THE COMMUNITY?

GEOGRAPHICAL DIVISIONS	YES	NO	THINK SO	NOT	QUESTIONABLE	ALL CITIES
North Central	180	31	28	11	17	267
North Atlantic	157	20	22	9	17	225
Western	40	7	3	2	2	54
South Central	61	3	10	1	1	76
South Atlantic	26	3	2	0	1	32
Counties	87	9	6	4	2	108
	551	73	71	27	40	762

TABLE LIV

(Reduced to per cents)

GEOGRAPHICAL DIVISIONS	YES	NO	THINK SO	NOT	QUESTIONABLE	ALL CITIES
North Central	67.4	11.7	10.4	4.1	6.4	100
North Atlantic	67.7	8.9	9.7	4.0	7.7	100
Western	74.1	13.0	5.5	3.7	3.7	100
South Central	80.2	4.0	13.2	1.3	1.3	100
South Atlantic	81.3	9.3	6.3	0.0	3.1	100
Counties	80.5	8.3	5.6	3.7	1.9	100
	72.3	9.6	9.3	3.5	5.3	100

Important variations in practice are not discoverable until the replies of superintendents are distributed for the different-sized cities, irrespective of their location. But when this is done, it is not clear that one group of cities is more progressive or conservative than another group of cities. It is true that variations occur, but it seems that superintendents of small cities are no more likely to maintain that the proper way of increasing

efficiency in arithmetic is by increasing the social character of the material than are superintendents of large cities. At any rate these tables are descriptive of a tendency that is characteristic of every section of the United States and of superintendents of every sized city.

TABLE LV

CAN TIME BE SAVED BY RELATING PROBLEMS TO INDUSTRIES, OCCUPATIONS, ETC., OF THE COMMUNITY?

SIZE OF CITY	YES	NO	THINK SO	NOT	QUESTIONABLE	ALL CITIES
1,000,000 and over	3	0	0	0	0	3
200,000 to 999,999	9	4	2	0	1	16
100,000 to 199,999	6	2	0	1	1	10
50,000 to 99,999	24	3	2	1	2	32
30,000 to 49,999	24	2	6	2	3	37
20,000 to 29,999	34	5	2	2	3	46
15,000 to 19,999	29	7	3	2	1	42
10,000 to 14,999	69	9	5	3	4	90
8,000 to 9,999	57	6	10	3	8	84
4,000 to 7,999	209	26	35	9	15	294
	464	64	65	23	38	554

TABLE LVI

(Reduced to per cents)

SIZE OF CITY	YES	NO	THINK SO	NOT	QUESTIONABLE
1,000,000 and over	100.0	0.0	0.0	0.0	0.0
200,000 to 999,999	56.2	25.0	12.5	0.0	6.3
100,000 to 199,999	60.0	20.0	0.0	10.0	10.0
50,000 to 99,999	75.0	9.4	6.2	3.2	6.2
30,000 to 49,999	65.0	5.4	16.2	5.4	8.0
20,000 to 29,999	74.0	10.8	4.3	4.3	6.6
15,000 to 19,999	69.5	16.6	7.1	4.7	2.3
10,000 to 14,999	76.7	10.0	5.5	3.4	4.4
8,000 to 9,999	67.8	7.2	11.8	3.6	9.6
4,000 to 7,999	71.1	8.8	11.9	3.1	5.1
	70.9	9.7	9.9	3.5	6.0

The Social Character of the Material in the Indianapolis Course of Study

Many courses of study show a tendency on the part of supervisors to reconstruct the course in arithmetic in accordance with social and business demands. One of the most noteworthy of these is the Indianapolis course of study in which there is a special attempt at supplementing the arithmetic of the seventh and eighth grades with community problems. The problems outlined for the 7 A grade deal with the grocery, meat market, the department store, lumber dealers, the cost of heating and lighting the home, the cost of furnishings for the home, saving money, banking, interest, real estate, and the loaning of money. In addition to these the eighth-grade list includes problems referring to the dairy and milk department, fire department, city market, city hospital, library, street construction, transportation, insurance, stocks and bonds. In every instance these problems are directly related to local conditions. It could not, however, be maintained that the training that the student is getting in the solution of these problems would be of no service to him elsewhere, as Indianapolis is a typical American city. The cost of food, clothing, and furnishings, and the agencies that may be used for the saving of money are very much the same everywhere.

Indianapolis authorities have merely taken advantage

of local conditions for the purpose of teaching arithmetic facts and problems that people universally need. Such a device is calculated not only to increase the interest of students in arithmetic, but to insure the fixation of certain fundamental phases of the subject. The students are encouraged to make original problems and to observe that arithmetic plays an important part in their daily life and in that of the community in general. We are of the opinion that such a device as this economizes time and energy, because it rationalizes the processes and because the students are immediately conscious of the recurrence of their arithmetical experiences in the world outside.

CHAPTER XII

TESTS AND RESULTS AS SHOWN BY SPECIAL INVESTIGATIONS

A Comparison of the Arithmetical Efficiency of To-day with that of the Past

THE SPRINGFIELD TESTS

Within recent years there has been much discussion in regard to the results attained under present-day conditions in the teaching of arithmetic as compared with the work of earlier days. For the most part the discussion has confined itself to statements of opinion or proof by isolated examples.

However, if a supervisor wishes to compare the work of a particular school of to-day with that of a particular school of the past, means are available.

A few years ago a set of examination questions with the papers and their markings, which were given in the Springfield, Massachusetts, schools in 1846, were discovered. The examination was given to eighty-five pupils in the ninth grade. The average marking of the class of 1846 was 29.4 per cent. In 1905 the eighth-grade class in Springfield, Massachusetts, averaged 65.5

per cent. The Springfield questions in arithmetic have been given in many cities since they were brought to light. In practically all instances the children of to-day have attained a higher rank than the children in 1846. In Frankfort, Indiana, the class averaged 62.2 per cent; in Albia, Iowa, the class averaged 74.5 per cent. The questions are as follows:

1. Add together the following numbers: Three thousand and nine, twenty-nine, one, three hundred and one, sixty-one, sixteen, seven hundred and two, nine thousand, nineteen and a half, one and a half.

2. Multiply 10,008 by 8009.

3. In a town five miles wide and six miles long, how many acres?

4. How many steps of two and a half feet each will a person take in walking one mile?

5. What is one third of $175\frac{1}{2}$?

6. A boy bought three dozen of oranges for $37\frac{1}{2}$ cents, and sold them for $1\frac{1}{2}$ cents apiece. What would he have gained if he had sold them for $2\frac{1}{2}$ cents apiece?

7. There is a certain number, one third of which exceeds one fourth of it by two. What is the number?

8. What is the simple interest of $ 1200 for 12 years, 11 months, and 29 days, at six per cent?

THE PIONEER INVESTIGATION

In 1902, Dr. J. M. Rice,[1] who had earlier interested himself in testing the results secured in other subjects,

[1] J. M. Rice, *Scientific Management in Education.* Hinds, Noble, & Eldredge. New York, 1903.

gave a test to 6000 grammar-grade children distributed through eighteen school buildings in seven cities. He examined each of the pupils in the fourth, fifth, sixth, seventh, and eighth grades by means of eight examples. Concerning these examples, Dr. Rice says, "In preparing my questions, I endeavored to arrange them in a way that would suit the individual grades of all schools, regardless of the methods or systems employed. From this standpoint I was successful, excepting that in a very few instances two of the examples were beyond the scope of the pupils in the first half of the fourth year, because they had not yet learned to multiply or divide with figures above twelve, and in the first half of the seventh year, where the classes had not yet had much practice in percentage. These points were carefully noted; but when the papers were marked, it was found that the effect upon the entire school average would not in any case exceed 2 per cent. I wish to add, furthermore, that for the purpose of studying the growth of mental power from year to year, some of the problems were carried through several grades. Thus, of the eight questions for the fourth grade, five were repeated in the fifth and three in the sixth, etc. Moreover, this repetition will enable us to see not only, for instance, how the results in the fifth and sixth grades, in regard to certain problems, compare with those of the fourth in the same school, but also how the results in the fourth grade of some

schools compare in these examples with those of the later grades of others, etc.

"The problems for each were as follows:

FOURTH YEAR

1. A man bought a lot of land for $ 1743, and built upon it a house costing $ 5482. He sold them both for $ 10,000. How much money did he make?

2. If a boy pays $2.83 for a hundred papers, and sells them at 4 cents apiece, how much money does he make?

3. If there were 4839 classrooms in New York City, and 47 children in each classroom, how many children would there be in the New York schools?

4. A man bought a farm for $ 16,575, paying $ 85 an acre. How many acres were there in the farm?

5. What will 24 quarts of cream cost at $ 1.20 a gallon?

6. A lady bought 4 pounds of coffee at 27 cents a pound, 16 pounds of flour at 4 cents a pound, 15 pounds of sugar at 6 cents a pound, and a basket of peaches for 95 cents. She handed the storekeeper a $ 10 note. How much change did she receive?

7. I have $ 9786. How much more must I have in order to be able to pay for a farm worth $ 17,225?

8. If I buy 8 dozen pencils at 37 cents a dozen, and sell them at 5 cents apiece, how much money do I make?

FIFTH YEAR

1. A man bought a lot of land for $ 1743, and built upon it a house costing $ 5482. He sold them both together for $ 10,000. How much did he make?

2. If a boy pays $ 2.83 for a hundred papers, and sells them at 4 cents apiece, how much does he make?

3. What will 24 quarts of cream cost at $1.20 a gallon?

4. If I buy 8 dozen pencils at 37 cents a dozen, and sell them at 5 cents apiece, how much money do I make?

5. A flour merchant bought 1437 barrels of flour at $7 a barrel. He sold 900 of these barrels at $9 a barrel, and the remainder at $6 a barrel. How much did he make?

6. How many feet long is a telegraph wire extending from New York to New Haven, a distance of 74 miles? There are 5280 feet in a mile.

7. A merchant bought 15 pieces of cloth, each containing 62 yards. He sold 234 yards. How many dress patterns of 12 yards each did he have left?

8. Frank had $3.08. He spent $\frac{1}{4}$ of it for a cap, $\frac{1}{7}$ of it for a ball, and with the remainder bought a book. How much did the book cost?

SIXTH YEAR

1. If a boy pays $2.83 for a hundred papers, and sells them at 4 cents apiece, how much does he make?

2. What will 24 quarts of cream cost at $1.20 a gallon?

3. If I buy 8 dozen pencils at 37 cents a dozen, and sell them at 5 cents apiece, how much do I make?

4. A flour merchant bought 1437 barrels of flour at $7 a barrel. He sold 900 of these barrels at $9 a barrel, and the remainder at $6 a barrel. How much did he make?

5. If a train runs $31\frac{2}{3}$ miles an hour, how long will it take the train to run from Buffalo to Omaha, a distance of 1045 miles?

6. If a map 10 inches wide and 16 inches long is made on a scale of 50 miles to the inch, what is the area in square miles that the map represents?

7. The salt water which was obtained from the bottom of a mine of rock salt contained 0.08 of its weight of pure salt. What

weight of salt water was it necessary to evaporate in order to obtain 3896 pounds of salt?

8. A gentleman gave away $\frac{1}{7}$ of the books in his library, lent $\frac{1}{6}$ of the remainder, and sold $\frac{1}{5}$ of what was left. He then had 420 books remaining. How many had he at first?

SEVENTH YEAR

1. If a map 10 inches wide and 16 inches long is made on a scale of 50 miles to the inch, what is the area in square miles that the map represents?

2. The salt water which was obtained from the bottom of a mine of rock salt contained 0.08 of its weight of pure salt. What weight of salt water was it necessary to evaporate in order to obtain 3896 pounds of salt?

3. A gentleman gave away $\frac{1}{7}$ of the books in his library, lent $\frac{1}{6}$ of the remainder, and sold $\frac{1}{5}$ of what was left. He then had 420 books remaining. How many had he at first?

4. A farmer's wife bought 2.75 yards of table linen at $0.87 a yard and 16 yards of flannel at $0.55 a yard. She paid in butter at $0.27 a pound. How many pounds of butter was she obligated to give?

5. If coffee sold at 33 cents a pound gives a profit of 10 per cent, what per cent of profit would there be if it were sold at 36 cents a pound?

6. Sold steel at $27.60 a ton, with a profit of 15 per cent, and a total profit of $184.50. What quantity was sold?

7. If a woman can weave 1 inch of rag carpet a yard wide in 4 minutes, how many hours will she be obligated to work in order to weave the carpet for a room 24 feet long and 24 feet wide? No deduction is to be made for waste.

8. A fruit dealer bought 300 apples at the rate of 5 for a cent, and 300 at 4 for a cent. He sold them at the rate of 8 for 5 cents. What per cent did he gain on the investment?

EIGHTH YEAR

1. If a map 10 inches wide and 16 inches long is made on a scale of 50 miles to the inch, what is the area in square miles that the map represents?

2. The salt water which was obtained from the bottom of a mine of rock salt contained 0.08 of its weight of pure salt. What weight of salt water was it necessary to evaporate in order to obtain 3896 pounds of salt?

3. A gentleman gave away $\frac{1}{7}$ of the books in his library, lent $\frac{1}{6}$ of the remainder, and sold $\frac{1}{5}$ of what was left. He then had 420 books remaining. How many had he at first?

4. A man sold 50 horses at $ 126.00 each. On one half of them he made 20 per cent, and on the other half he lost 10 per cent. How much did he gain?

5. Sold steel at $ 27.60 a ton, with a profit of 15 per cent and a total profit of $ 184.50. What quantity was sold?

6. A fruit dealer bought 300 apples at the rate of 5 for a cent, and 300 at 4 for a cent. He sold them at the rate of 8 for 5 cents. What per cent did he gain on his investment?

7. The insurance of $\frac{2}{3}$ of the value of a hotel and furniture cost $420.00. The rate being 70 cents on $100.00, what was the value of the property?

8. Gunpowder is composed of nitre 15 parts, charcoal 3 parts, and sulphur 2 parts. How much of each in 360 pounds of gunpowder?

The average for each city is presented in the following distribution:

	Grades					School Average	
	IV	V	VI	VII	VIII		
	Results	Results	Results	Results	Results	Results	Minutes Daily
City III	68.4	79.5	79.3	81.1	91.7	80.0	53
City I	72.7	84.7	80.4	64.2	80.9	76.6	60
City I	—	80.3	80.9	43.5	72.7	69.3	45
City I	54.5	74.7	72.2	63.5	74.5	67.8	45
City I	60.0	70.8	69.6	54.6	66.5	64.3	45
City II	81.3	78.2	71.2	33.6	36.8	60.2	60
City III	70.1	53.6	43.7	53.9	51.1	54.5	60
City IV	70.5	73.2	58.9	31.2	41.6	55.1	60
City IV	62.9	70.5	59.8	—	22.5	53.9	—
City IV	59.8	65.3	54.9	35.2	43.5	51.5	60
City IV	53.5	53.5	42.3	16.1	48.7	42.8	—
City V	38.5	67.0	44.1	29.2	51.1	45.9	40
City VI	28.1	38.1	68.3	33.5	26.9	39.0	33
City VI	41.6	45.3	46.1	19.5	30.2	36.5	30
City VI	36.8	55.0	34.5	30.5	23.3	36.0	48
City VII	59.3	53.7	35.2	29.1	25.1	40.5	42
City VII	47.4	65.4	35.2	15.0	19.6	36.5	45
City VII	41.1	37.5	27.6	8.9	11.3	25.3	45
General Average .	59.5	69.4	60.7	39.4	49.4	55.7	—
Number of pupils examined . .						Total,	5963

Mr. Rice found great variation in the reaction of pupils to these questions. The variability was even greater in the advanced grades than in the earlier grades. In the seventh grade the class averages ranged from as low as 8.9 per cent to as high as 81.1 per cent, and in the eighth grade the range was from 11.3 per cent to 91.7 per cent. Not only were the extremes

widely separated, but "the averages for schools taken as a whole varied between 25 and 80 per cent."

While it is true that wide variation was found in each school, yet Mr. Rice found that the performance of the different schools in the same city were highly similar; that is to say, if one school in a system ranked high, all the other schools in the system showed a similar ranking, and if one school ranked low, the system probably ranked low. Concerning the causes of the differences in performance, Mr. Rice proposes that "two factors must be taken into consideration: first, the influence of the teaching; and, secondly, the resistance against that influence due to circumstances over which the teacher has no direct control. . . . Analysis of the problem will show that the essential elements of which it is composed (resistance) do not exceed three in number: (1) the home environment of the pupils; (2) the size of the classes; and (3) the average age of the children."

After Mr. Rice's detailed study of the specific effect of each of these factors, he says: "A study of the figures . . . will show conclusively that the influence of all these factors has been very much exaggerated, and, therefore, that the cause of unfavorable results must be sought, largely at least, on the pedagogical side. . . . That is to say, the school laboring under the poorest conditions in respect to home environment obtained a better standing than any one of the so-called aristocratic schools. . . .

Equally surprising, if indeed not more incredible, may appear the statement that no allowance whatever is to be made for the size of the class in judging the results of my tests. . . . The number of pupils per class was larger in the highest six schools than it was in the schools of City VI, and the classes were exceptionally small in the school that stands at the lower end. . . . A glance at the ages will show that the average age of the pupils of the schools that showed the best results was about five months higher than that of the pupils of the schools that did the poorest. . . . But the factor of age may be completely eliminated by comparing the results of a given grade of the successful schools with those of a higher grade of the unsuccessful ones. . . . These facts certainly constitute a striking blow at the theory of those who believe that arithmetic is a matter of natural evolution."

Mr. Rice, in the further analysis of the factors affecting the efficiency of the arithmetical performances of the children on the basis of the time expenditure, stated the following conclusions: "A glance at the figures will tell us at once that there is no direct relation between time and results; that special pressure does not necessarily lead to success, and conversely, that lack of pressure does not necessarily mean failure.

"In the first place, it is interesting to note that the amount of time devoted to arithmetic in the school that

obtained the lowest average, 25 per cent, was practically the same as it was in the one where the highest average, 80 per cent, was obtained. From these few facts two important deductions may be made: first, that the unsatisfactory results cannot be accounted for on the ground of insufficient instruction; and, secondly, that the school showing the favorable results cannot be accused of having made a fetich of arithmetic."

The supervisor is especially interested in the analysis which Mr. Rice makes of the pedagogical aspects of the problem. He discusses this from two points of view: first, the part played by the teacher; and secondly, the part played by the supervisor. Concerning the former he says: "The elements brought into play by the teacher, though numerous, may be, for practical purposes, resolved into three primary factors:

"1. The time devoted to arithmetic;

"2. The methods of instruction; and

"3. Teaching ability, as represented by a combination of education, training, and the personality of the teacher."

Concerning the time element, Mr. Rice, in addition to the results presented earlier in this chapter, says: "In view of the results and of my interview with principals and teachers, I feel confident that home-work in arithmetic means a tax upon the time and energy of the pupil, for which he receives very meager, if any,

compensation. Consequently, I wish to add to my suggestions, as to the amount of time to be apportioned to arithmetic, that the forty-five minutes daily should stand for the preparation and recitation combined.

"Secondly, methods of teaching can certainly not be looked upon as the controlling element. . . . In the schools that passed my test satisfactorily no special methods had been in use. . . . Thoroughness is, undoubtedly, one of the secrets of success.

"Variations in the results cannot be accounted for by the differences in the general qualities of the teacher. Few will take exception to the statement that marked individual variations will be found among the members of every corps of teachers. Therefore, if general ability were the controlling factor, the extreme variations in results should be found in the different classrooms of the same locality. But this condition does not appear in the table, where it is shown that in certain localities practically all the results were good, while in certain other cities practically all the results were poor."

Thus Mr. Rice disposes of the factors of time, methods of instruction, and special teaching ability; his conclusion being that the variability in results does not parallel variability in these factors. He next attempts to analyze the results of the test with a view of determining the effect of their supervisory factors. Concerning this he says: "The facts have led me to believe that it is here that the controlling factor lies. My conviction is based on the circumstance that, in every instance, a variation in the results appears to accord

with a variation in a special phase of the supervision. If my interpretation of the facts is correct, we are forced to conclude that the results secured in the average classroom do not represent the powers of the average teacher, but the response to what is expected of her; so that, ultimately, the problem of results becomes a question of demand and supply. And my deduction is this, that the teachers will supply what their supervisors demand, provided the demand be placed within reasonable bounds."

Mr. Rice differentiated the functions of the superintendent under five heads:

1. The preparation of the course of study;
2. The apportionment of time to the individual subjects;
3. Offering suggestions to teachers, during meetings and visits, as to methods of teaching and the treatment of children;
4. The establishment of demands in regard to results; and
5. The testing for results to see whether the teachers are living up to these demands.

After analyzing the varying factors, Mr. Rice made the following generalization: "The controlling factor in the accomplishment of results is to be found in the system of examinations employed, some systems leading to better results than others. . . . The controlling element lies, therefore, in that form of examination which is intended as a test of the teacher's progress." These tests were classified as follows:

1. Tests made from time to time by the teachers themselves. Each teacher formulates her own questions, marks the papers of her own class, and submits the results to the superintendent; but no tests are made by principal or superintendent.

2. Tests made in the same way by the teachers; but the teachers' tests are supplemented from time to time by those of the superintendent.

3. Tests made from time to time by the principals, each principal formulating the questions for his own school. The results are reported to the superintendent, but the latter does not make any tests of his own.

4. The same system of testing by the principals; but the principals' tests are supplemented from time to time by those of the superintendent.

Of those foregoing classes of tests, Mr. Rice selects the fourth as being the system best calculated to attain a high standard of efficiency.

The supervisor of the teaching of arithmetic finds much in the investigation of Mr. Rice to encourage closer supervision in the teaching of arithmetic. His conclusions to the effect that results are more directly traceable to the supervisor's activity than to any other one cause, are of great significance not only to the superintendent but also to the teacher. Supervisors will do well to experiment definitely with the different types of tests which Mr. Rice proposes. It may be readily seen that one of the most difficult problems is that of determining the standard of results which are to be demanded of children in the different grades.

Abilities of Children in the 6 A Grade in the Fundamentals and also in Reasoning

Dr. C. W. Stone[1] in 1908 gave the same test under as nearly uniform conditions as possible to children in the 6 A grade distributed throughout twenty-six different cities in the United States. Some of the exercises were so planned as to test the ability of children in the fundamental operations. The other part of the examination tested the reasoning ability of the children.

A time limit of twelve minutes was given for the solution of the fundamental problems, and fifteen minutes for the reasoning problems. The list of problems in each case was so long as to make it impossible for any student to solve all of them in the time given.

Each child had a chance to do all he could in the time allotted. Ordinarily, examinations are arranged in such a way as to enable the bright pupil to finish his work before the time is up, and as a consequence it is never known just how much more that pupil could have done in the time allotted had he worked at his maximum speed. On the other hand, the slow pupil finds it impossible to finish. In other words, the ordinary examination tends to conceal the extreme range of variation which actually exists in the ordinary class. It is as though we were to announce that every one of a

[1] C. W. Stone. "Arithmetical Abilities and Some Factors Determining Them." *Teachers College Record*. New York City. 1910.

given group should run a hundred yards in fourteen seconds. Swift runners would cover the distance in less than fourteen seconds, but slow runners would require a longer time.

Dr. Stone scored all the papers alike. The reliability of system of scoring used was checked up by a preliminary test.

Interest at once attaches to the results attained by the 6 A children in the different cities. The fact that children in the same grade, distributed throughout the different cities, were tested by the same person, using the same questions, with the same time limit, and the same system of scoring, enables Dr. Stone to compare the arithmetical ability of the children in the different cities.

It also enabled him to compare the results in one city with those in another city. Comparisons of this sort are of very great value for supervisory purposes. While they do not supply the supervisor with a remedy when he discovers that his schools are far above or far below the achievement of other schools, the comparisons at least reveal the place that requires a more careful study and diagnosis. As a matter of fact these comparisons did turn the attention of Dr. Stone to the study of the curriculum, methods of instruction, and supervisory helps for an explanation of the variation.

The following table shows the ranking of the different systems, the range of variability of the systems as a whole.

ACHIEVEMENTS OF THE SYSTEMS AS SYSTEMS MEASURED BY SCORES MADE

TABLE I

Scores of the Twenty-six Systems in Reasoning with Deviations from the Median. Scores from All Problems. M = 551.

Systems in Order of Achievement	Scores Made	Deviations from the Median	Deviations in Per Cent of the Median
XXIII	356	−195	−35
XXIV	429	−122	−22
XVII	444	−107	−19
IV	464	−87	−16
XXV	464	−87	−16
XXII	468	−83	−15
XVI	469	−82	−15
XX	491	−60	−11
XVIII	509	−42	−8
XV	532	−19	−3
III	533	−18	−3
VIII	538	−13	−2
VI	550	−1	−2
I	552	1	2
X	601	50	9
II	615	64	12
XXI	627	76	14
XIII	636	85	15
XIV	661	110	19
IX	691	140	20
VII	734	183	33
XII	736	185	34
XI	759	208	38
XXVI	791	240	44
XIX	848	297	54
V	914	363	66

TABLE II

Scores of the Twenty-six Systems in Fundamentals with Deviations from the Median. Scores from all Problems. M = 3111.

Systems in Order of Achievement	Scores Made	Deviations from the Median	Deviations in Per Cent of the Median
XXIII	1841	−1270	−41
XXV	2167	−944	−30
XX	2168	−943	−30
XXII	2311	−800	−26
VIII	2747	−364	−12
X	2749	−362	−12
XV	2779	−332	−11
III	2845	−266	−8
I	2935	−176	−6
XXI	2951	−160	−5
II	2958	−153	−5
XVII	3042	−69	−2
XIII	3049	−62	−2
VI	3173	62	2
XI	3261	150	5
IX	3404	293	9
XII	3410	299	10
XXIV	3513	402	13
XIV	3561	450	14
IV	3563	452	14
V	3569	458	15
XXVI	3682	571	18
XVI	3707	596	19
XVIII	3758	647	21
VII	3782	671	22
XIX	4099	988	31

It is important to note the wide range of ability displayed. The supervisor is at once concerned as to the possible cause for the wide range of performance by children in the same grade. The supervisor asks, Is

this due to difference in time allotment, to difference in method, to difference in material, to difference in children, or to difference in supervision?

Dr. Stone distributed his data so as to show the relationship which existed between achievement in arithmetic and time allotment. His conclusion follows:

> For these systems it is evident that there is practically no relation between time expenditure and arithmetical abilities; and, in view of the representative nature of these twenty-six systems, it is probable that this lack of relationship is the rule the country over. This is not to say that a certain amount of time is not essential to the production of arithmetical abilities; nor that, given the same other factors, operating equally well, the product will not increase somewhat with an increased time expenditure. What is claimed is that, as present practice goes, a large amount of time spent on arithmetic is no guarantee of a high degree of efficiency. If one were to choose at random among the schools with more than the median time given to arithmetic, the chances are about equal that he would get a school with an inferior product, and conversely, if one were to choose among the schools with less than the median time cost, the chances are about equal that he would get a school with a superior product in arithmetic.

Dr. Stone also attempted to compare the rankings of the performance of the 6 A children with the rankings of the courses of study. So far as he was able to determine, there seemed to be little if any relation between these. This investigation corroborates that of Mr. Rice to the effect that the most important factor in

attaining efficiency in arithmetic is that of close supervision of the teacher.

With reference to the relationships existing between the different abilities measured, Dr. Stone says: "It seems safe to say tentatively of the fundamentals that the possession of ability in addition is the least guarantee of the possession of ability in others; that the possession of ability in multiplication is the best guarantee of the possession in others; and that this probably means that multiplication is like addition on its mechanical side and like division on its thinking side. Hence, if it is desired to measure abilities in fundamentals by a single test, one in multiplication would be best; and a test in division would probably be the best single measure of arithmetical ability." All of which tends to confirm the thesis proposed by Thorndike and others; namely, that there is no such thing as general ability in arithmetic. The fact that a child is able to do one thing in arithmetic is no guarantee that he can do other things that are apparently very similar. In other words, a supervisor of the teaching of arithmetic cannot be satisfied until he knows the degree of proficiency which each child possesses in each of the desired arithmetical abilities.

Many supervisors have been interested in applying the same tests to their own 6 A grade children. For the benefit of others, who may wish to try this experiment,

we print the questions used in this investigation. For the close student of the problem nothing short of a detailed study of this experiment will suffice.

THE TESTS AS GIVEN

Work as many of these problems as you have time for; work them in order as numbered:

1. Add

$$\begin{array}{r} 2375 \\ 4052 \\ 6354 \\ 260 \\ 5041 \\ \underline{1543} \end{array}$$

2. Multiply 3265 by 20.
3. Divide 3328 by 64.
4. Add

$$\begin{array}{r} 596 \\ 428 \\ 94 \\ 75 \\ 302 \\ 645 \\ 984 \\ \underline{897} \end{array}$$

5. Multiply 768 by 604.
6. Divide 1918962 by 543.
7. Add

$$\begin{array}{r} 4695 \\ 872 \\ 7948 \\ 6786 \\ 567 \\ 858 \\ 9447 \\ 7499 \end{array}$$

8. Multiply 976 by 87.

9. Divide 2782542 by 679.

10. Multiply 5489 by 9876.

11. Divide 5099941 by 749.

12. Multiply 876 by 79.

13. Divide 62693256 by 859.

14. Multiply 96879 by 896.

Solve as many of the following problems as you have time for; work them in order as numbered:

1. If you buy 2 tablets at 7 cents each and a book for 65 cents, how much change should you receive from a two-dollar bill?

2. John sold 4 Saturday Evening Posts at 5 cents each. He kept $\frac{1}{2}$ the money and with the other $\frac{1}{2}$ he bought Sunday papers at 2 cents each. How many did he buy?

3. If James had 4 times as much money as George, he would have $ 16. How much money has George?

4. How many pencils can you buy for 50 cents at the rate of 2 for 5 cents?

5. The uniforms for a baseball nine cost $ 2.50 each. The shoes cost $ 2 a pair. What was the total cost of uniforms and shoes for the nine?

6. In the schools of a certain city there are 2200 pupils; $\frac{1}{2}$ are in the primary grades, $\frac{1}{4}$ in the grammar grades, $\frac{1}{8}$ in the High School and the rest in the night school. How many pupils are there in the night school?

7. If $3\frac{1}{2}$ tons of coal cost $ 21, what will $5\frac{1}{2}$ tons cost?

8. A newsdealer bought some magazines for $ 1. He sold them for $ 1.20, gaining 5 cents on each magazine. How many magazines were there?

9. A girl spent $\frac{1}{8}$ of her money for car fare, and three times as much for clothes. Half of what she had left was 80 cents. How much money did she have at first?

10. Two girls receive \$2.10 for making buttonholes. One makes 42, and the other 28. How shall they divide the money?

11. Mr. Brown paid one third of the cost of a building; Mr. Johnson paid $\frac{1}{2}$ the cost. Mr. Johnson received \$500 more annual rent than Mr. Brown. How much did each receive?

12. A freight train left Albany for New York at 6 o'clock. An express left on the same track at 8 o'clock. It went at the rate of 40 miles an hour. At what time of day will it overtake the freight train if the freight train stops after it has gone 56 miles?

Additional Tests for Fundamentals and Reasoning

Superintendent Giles[1] of Richmond, Indiana, prepared a series of simple tests or formulæ by which any teacher might at any time measure, within reasonable limits, the ability of any school or pupil in the fundamentals of arithmetic and in reasoning power with abstract numbers. The object of the tests or formulæ, according to Superintendent Giles, is to determine the average percentage of accuracy, together with a measure of the variation in the various fundamental operations and in reasoning.

The conditions under which the proposed tests are to be given are as follows:

[1] J. T. Giles, "The Scientific Study of Arithmetic Work in School." N. E. A. 1912, pp. 488–492.

1. Time for each test, from four to five minutes.

2. The tests are to be previously written on the blackboard, plainly, in a good light, and covered by a screen or curtain until the tests begin.

3. The tests are to be arranged in rows and columns, each row containing ten problems and a sufficient number of rows given to supply more problems than can be worked by any pupil in the time allowed.

4. Pupils write on papers previously prepared in cross-sections ten wide to correspond with the arrangement of problems on the board.

5. All pupils to begin writing, giving answers only, as soon as the curtain is drawn and continue until curtain falls.

6. The numbers composing the test to be drawn by the teacher by lot in the following manner: Fifty strips of paper are prepared of uniform size, on each of which is written at regular intervals the natural series of digits omitting 0 and 1. These strips are then cut into pieces of uniform size, each containing a digit of the series 2–9. These digits are then placed in a bag and drawn one by one, as needed in formulating the test. This arrangement insures an equal number of each of the digits to draw from. Zero and 1 are omitted to avoid the possibility of chance combinations of exceptional ease of solution. Tests four minutes long and over, formed in this way, would vary but slightly in the degree of difficulty.

7. Where a chance drawing would result in an absurdity, as in the subtraction of a larger from a smaller number, the order of the last two digits drawn should be reversed.

8. The operation to be tested in each exercise is to be explained to the class by the teacher before drawing the curtain. It should also be written above the test as well as indicated by placing the digits in the position usually adopted for performing the various

operations, *i.e.*, one above the other, with a line below in addition, subtraction, and multiplication and the usual arrangement for short division, omitting all the operation signs $+$, $-$, $\times$, $\div$.

9. Pupils may grade and mark their own papers, which should be checked later by the teacher. From these marks the average or median accomplishment of the school can be quickly obtained and a measure of the variation easily derived.

The subject matter of the proposed tests is as follows:

(1) Addition in single combinations, also double column with two figures in column.

(2) The same arrangement for subtraction.

(3) Multiplication in which both factors are single digits, also when the multiplicand is composed of two digits.

(4) Short division in which the divisor is a single digit and the dividend two, also with three digits in the dividend.

The reasoning tests include (1) three-quantity one-step abstract problems and (2) four-quantity two-step problems.

The formulæ may be expressed by letters which are to be replaced with the digits drawn from the bag. Two letters written together mean a number of two digits, not multiplication as in algebra.

For addition we have:

Find the sum:

$$\begin{array}{cccc} a & c & e & g \\ \underline{b} & \underline{d} & \underline{f} & \underline{h} \end{array} \quad \text{etc.}$$

Also

$$\begin{array}{ccc} ab & ef & ij \\ \underline{cd} & \underline{gh} & \underline{kl} \end{array} \quad \text{etc.}$$

For subtraction:

Find the difference:

$$\begin{array}{ccccc} a & c & e & g & i \\ \underline{b} & \underline{d} & \underline{f} & h & j \end{array} \quad \text{etc.}$$

$$\begin{array}{lll} ab & ef & ij \\ \underline{cd} & \underline{gh} & \underline{kl} \end{array}$$

Multiplication:

Write products:

$$\begin{array}{lllll} a & c & e & g & \\ \underline{b} & \underline{d} & \underline{f} & \underline{h} & \text{etc.} \end{array}$$

$$\begin{array}{llll} ab & de & gh & \\ \underline{\ c\ } & \underline{\ f\ } & \underline{\ i\ } & \text{etc.} \end{array}$$

Division:

$a)bc$ $d)ef$ $g)hi$, etc.
$a)bcd$ $e)fgh$ $i)jkl$, etc.

One-step reasoning formulæ:

$$a + b = c \qquad a \times b = c \qquad a \div b = c$$

from which are derived the following problems:

Indicate by the abbreviation add., sub., mul., or div. the operation to be performed in the two given numbers to get the required one.

1. The sum of two numbers is a. One is b, what is the other?
2. The difference of two numbers is a. One is b, what is the other?
3. The product of two numbers is a. One is b, what is the other?
4. The quotient of two numbers is a. One is b, what is the other?
5. What number added to b gives c?
6. What number subtracted from b gives c?
7. What number multiplied by b gives c?
8. What number divided by b gives c?
9. a added to what number gives c?

10. a subtracted from what number gives c?

11. a multiplied by what number gives c?

12. a divided by what number gives c?

13. a is b more than what number?

14. a is b less than what number?

15. a is b times what number?

16. a is b divided by what number?

The problems in the two-step reasoning test are derived from the formulæ:

$$a + b = c - d$$
$$ab = cd$$
$$(a + b) = cd$$

1. What number added to a is equal to the sum of c and d?

2. What number subtracted from a is equal to the sum of c and d?

3. What number added to a is equal to the difference of c and d?

4. What number subtracted from a is equal to the difference of c and d?

5. What number multiplied by a is equal to the product of c and d?

6. What number multiplied by a is equal to the sum of c and d?

7. What number multiplied by a is equal to the difference of c and d?

8. What number added to a is equal to the product of c and d?

9. What number subtracted from a is equal to the product of c and d?

10. What number multiplied by the sum of a and b equals c?

11. What number multiplied by the difference of a and b equals c?

12. What number divided into the sum of a and b equals c?

13. What number divided into the difference of a and b equals c?

The order in which the reasoning problems are given is also to be determined by lot, two or more sets being used if necessary to provide sufficient problems for the time allowed.

It will be observed that the tests are inexpensive, easily manipulated, and readily interpreted. As yet, however, they are of little value for comparing one school or one grade with another, for they have not been adequately standardized. Their ultimate standardization will require that a number of superintendents in different parts of the country coöperate in giving them and in sending the results to some common clearing house for tabulation. The standards derived may then be utilized in determining the relative station of any individual, class, or school. It will be better, and in the long run far more economical, for tests of this sort to receive a universal trial than for a multitude of new ones to be constructed. We can hasten the day when definite standards will supplant opinion by duplicating the work of others in enough places to make the results universally valid.

Experiment on Way of Gaining Facility in the Use of the Multiplication Tables

Supervisors have been more or less seriously interested in a more economical way of teaching children the multiplication tables. Many ways have been proposed,

varying from that of having the children "stay in" after school in order to write the tables a given number of times, to that of teaching the tables purely incidentally. There have been very few experiments performed calculated to throw any light on this problem.

Mr. E. A. Kirkpatrick of the State Normal School at Fitchburg, Massachusetts, published[1] in 1914 an interesting experiment bearing upon this problem. In his study he recognized three characteristic ways of learning the tables: first, having the children memorize the multiplication tables in the traditional way; second, having the children placed in possession of a multiplication sheet which they could use as they needed, with the hope that after using it awhile, they would remember the combinations well enough so that they would be able to "work" the problems without the use of the sheet. He proposed as a third method that each child be taught to derive multiplication combinations from a knowledge of the combinations in addition, — "figure out each combination as he needed it."

As Mr. Kirkpatrick dealt with students in the Normal School who already knew the multiplication tables, he experimented upon the different ways of learning a new set of products, "of seven by the prime numbers from 17 to 53, inclusive, and the experiment was conducted

[1] E. A. Kirkpatrick, "An Experiment on Memorizing *vs.* Incidental Learning." *Journal of Educational Psychology*, Vol. V., pp. 405–413.

to determine the relative advantage of three methods of learning these combinations. A practice sheet or test sheet was prepared containing the prime numbers above named with a smaller figure seven beneath them arranged in 10 lines of 10 such groups of figures each. Another sheet, known as the key sheet, indicated the products that could be substituted for each group of numbers. In two methods of testing the pupils were not informed that the numbers on the key sheet were products, and the majority of them did not discover the fact, and very few of those who did made use of their knowledge."

Mr. Kirkpatrick then experimented upon two classes of young men in the normal school. After a preliminary test, one group practiced while the other group memorized for four or five days and then began practicing, using the key sheet if they needed it.

On the tenth day of the experiment, the fifth or sixth day of practice for the memorizers, their record of 17.2 seconds was slower than the fifth or sixth grade of practice for the practicers by 3.1 seconds.

After an interval of about three weeks, during which nothing was done with the experiment, a final test was given. In this test they wrote as many products as possible without the key sheet in two minutes. The memory group wrote on an average of 40.9 answers, and the practicing group 46.2. It appears, therefore, that from every point of view the results averaged better for the practice group than for the memory group.

The next test was tried with two classes of freshmen normal

school students of about 25 each, all of them young ladies except four. One class practiced, using the key as previously described, for eight days, while the other had no key, but multiplied the numbers and wrote the answers as rapidly as possible the same number of days. In the final test the average number of answers written in two minutes by the group practicing with the key was 25.4, while the computing group wrote 44.3 answers. The best of those practicing with the key were nearly, but not quite, as good as the best in the computing group, but only one wrote fifty or more answers and six not over twenty, while in the group that computed seven wrote fifty or more and only one less than twenty. Those who had memorized the key were helpless in the final efficiency test. Only two or three knew they could get the answers by multiplying. Those in the computing group were perfectly at home and only a little slower than those who incidentally memorized the final products.

Mr. Kirkpatrick experimented in a similar way with children, but owing to an epidemic, the conditions were not quite under control. However, Mr. Kirkpatrick made the following conclusions:

The result in the final efficiency test of two minutes was that the number of products written by the computers was 27.7; by the practicers, 19.1; by the fifth-grade memorizers, 10.1, and by the sixth-grade memorizers and practicers, 27.4.

While Mr. Kirkpatrick does not maintain that his experiments are conclusive, yet he does feel that they are highly suggestive. He says that "it seems that memorizing apart from use is the poorest method of all, drill in using somewhat better, while the method of using previous knowledge as a guide in practice is the

best of the three. The results indicate that in many lines of teaching there has been a tremendous waste of time, energy, and interest in first memorizing, then later practicing, the use of what has been learned. Also that pupils do better when practice is guided by their own knowledge than when it is directed by authority of book or teacher. It is probable that at least a year in numbers is wasted in special drill on the various combinations beyond what would be necessary if emphasis were first placed on learning how to compute tables, than upon working examples and problems with larger numbers, computing products till they were learned incidentally. This last statement has, of course, not been proved experimentally, but is merely an opinion based on inference from general principles and on a good deal of personal observation. Just how much time could be saved by the use of computation and incidental methods in number work as compared with memorizing and special drill methods is not known, but there can be no doubt that there is no need to have children memorize any tables."

Practice in the Case of School Children

Two experiments, one in addition and the other in division, were made by Dr. T. J. Kirby[1] with 1350

[1] T. J. Kirby. "Practice in the Case of School Children." *Teachers College Record*, Columbia University.

children in the schools of the Children's Aid Society, New York City. The work with addition was given to classes in the fourth year of the elementary schools; the division, to classes in the last half of the third year and the first half of the fourth year. Thorndike's "Addition Sheets" and the "Remainder Division Tables" were used as a basis for the practice. These addition sheets, seven in number, each contain 48 columns of one-place numbers so arranged that any successive five columns are of nearly equal difficulty. The division combinations used include the entire series from "20 = — 3's and — remainder" up to "89 = — 9's and — remainder," thus involving not only the combinations which are the inverse of the multiplication tables through 9 times 9, but also the division of the intervening series of numbers which give a remainder in the answer.

After an initial practice period of fifteen minutes, each class practiced the addition columns for forty-five minutes and then received a final test of fifteen minutes in length. The first and last practice or tests served as bases for determining the change in ability for each individual measured. The intervening practice of forty-five minutes between initial practice period and the final practice period was broken up for different groups of classes in four different ways. For Group I it was divided into two practice periods of $22\frac{1}{2}$ minutes each; for Group

II, into three practice periods of fifteen minutes each; and for Group III, into eight practice periods, seven of six minutes each and one of three minutes; for Group IV, into twenty-two practice periods, twenty-one of two minutes each and one of three minutes. The following table makes the plan clear:

GROUPS	INITIAL	INTERVENING 45 MINUTES		FINAL PERIOD
I	15 min.	2	22½ min.	15 min.
II	15 min.	3	15 min.	15 min.
III	15 min.	7	6 min. and 1 of 3 min.	15 min.
IV	15 min.	21	2 min. and 1 of 3 min.	15 min.

A similar plan was used in the division experiment. Each class had an initial practice period of ten minutes and a final period of ten minutes; and the intervening practice of 40 minutes was divided in three different ways. In the division, as in the addition, the initial and final practice periods were identical in character with the intervening practice periods; but besides serving as practice periods, they served as measures of ability at the beginning and end of practice from which any change in ability was measured. The first group had four intervening practice periods of twenty minutes each; a second group had four intervening practice periods of ten minutes each; a third group had twenty intervening practice periods of two minutes each. The following tabular statement presents the plan for each group:

Groups	Initial Period	Intervening 40 Minutes	Final Periods
I	10 min.	2 20 min.	10 min.
II	10 min.	4 10 min.	10 min.
III	10 min.	20 2 min.	10 min.

The pupils were encouraged to do their best, to work as fast as they could without making mistakes. Their papers were collected at the close of each practice and scored in a uniform manner, each column in addition and each quotient and remainder in division counting one. The conditions within the schoolroom and the time practice were kept as nearly uniform as possible from day to day.

Dr. Kirby's results show that the 732 fourth-grade children, who received practice in addition, had a median ability at the end of the first fifteen minutes of practice of 23.3 columns added correctly and a median accuracy of 79 per cent. In other words fifty per cent of the pupils added 23.3 per cent or more of the columns while fifty per cent added 23.3 per cent or less of them correctly. A median of 79 per cent of accuracy shows that there were as many children who added correctly four fifths or more of their problems as there were who added correctly four fifths or less. It will be observed that we have referred only to the problems worked correctly; the median number of problems actually worked was 29.5, the difference between the two medians being 6.2.

In the final fifteen minutes of practice in addition the

group added correctly a median of 10.7 more columns than in the initial fifteen minutes period. This meant a median percentile gain of 48 per cent for the practice. That speed and accuracy are not directly related is shown by the fact that this gain of 10.7 problems added correctly was accompanied by a median loss in accuracy of .4 per cent. This negative relation is more apparent than real, for the pupils actually tried more problems and solved more correctly during the last fifteen minutes of practice. They gained in speed, but while they were doing so their accuracy remained almost at a standstill.

The 606 children who took the practice in division showed an initial median ability of 34.5 combinations with a median per cent of accuracy of 93 per cent, and the final practice showed a median ability of 62 combinations with a median percentile gain in accuracy of 2.6 per cent.

We cannot be certain that Dr. Kirby's figures represent norms that may be applied to all third and fourth grade children, but they should be accepted until further experimentation refines them.

A fact of very great interest to every supervisor is the relative value of drill periods of different lengths. This was the particular problem with which Dr. Kirby was concerned. It will be remembered that he gave the practice in addition to four groups. His results in addition are summarized in the following tables:

MEDIANS OF THE GROUPS OF INDIVIDUALS

	Median Initial Ability. Examples Correct	Average Gross Gain	Median Gross Gain	Median Gain Per Cent	Median Gain in Accuracy
Group I . . .	22.9	11.0	9.5	45	3.5
Group II . . .	25.4	13.6	11.0	43	1.6
Group III . . .	21.1	10.7	9.6	42	1.5
Group IV . . .	25.1	16.1	12.6	56	2.7

In the above tables the individual in the group was considered as the unit; *i.e.*, these averages or medians represent central tendencies secured by tabulating the scores of all the pupils, irrespective of the classes in which they were registered. The table shows that the greatest gain in speed was made by the two-minute drill group, while the only group that made any gain in accuracy was the 22½-minute drill group.

The central tendency for each class was also computed and the average of these central tendencies was found. These averages corroborate the evidence presented in the preceding table.

AVERAGE OF CLASS MEDIAN

	Average Initial Median Ability of Classes. Examples Correct	Average Gross Median Gain of Classes	Average of Median Percentile Gains
Group I . . .	23.7	10.2	42
Group II . . .	25.7	9.6	41
Group III . . .	21.3	9.4	42
Group IV . . .	25.5	13.9	58

Using now the best measure of efficiency — the number of examples correct, which includes credit for both speed and accuracy — and the three methods of computing the gain, we have:

	Average Gross Gain of Individuals	Median Gross Gain of Individuals	Average of Median Gross Gain of Classes
Group I . . .	11.0	9.5	10.2
Group II . . .	13.6	11.0	9.6
Group III . . .	10.7	9.6	9.4
Group IV . . .	16.1	12.6	13.9

It therefore appears that, taking the results at their face value, the 2-minute practice produces the best results.

The summary in division is:

Medians of the Groups of Individuals

	Median Initial Ability	Average Gross Gain	Median Gross Gain	Median Gain Per Cent	Median Gain in Accuracy
Group I . . .	38.3	25.1	22.6	60	2.1
Group II . . .	32.0	25.5	23.5	73	3.5
Group III . . .	40.3	42.6	40.4	94	2.3

It will be observed that there was a marked difference in the initial ability of the three groups, with reference to the number of divisions made, although there seems to be little difference in their initial ability.

Group III gained almost twice as many combinations done correctly in the course of the practice as did the other two groups, whose gains were practically the same.

The above measures were computed from the scores of the individuals comprising these groups. The following measures are computed from the scores of the classes in the groups. The median for each class was found; then the average of these medians was computed.

AVERAGE OF CLASS MEDIANS

	AVERAGE INITIAL MEDIAN ABILITY OF CLASSES	AVERAGE GROSS MEDIAN OF CLASSES	AVERAGE OF MEDIAN PERCENTILE GAINS
Group I . . .	38.4	20.6	58
Group II . . .	33.4	25.1	77
Group III . . .	41.4	44.7	114

Using the three methods of computing gross gain, we have:

	AVERAGE GROSS GAIN OF INDIVIDUALS	MEDIAN GROSS GAIN OF INDIVIDUALS	AVERAGE OF MEDIAN GROSS GAINS OF CLASSES
Group I . . .	25.1	22.6	20.6
Group II . . .	25.5	23.5	25.1
Group III . . .	42.6	40.4	44.7

Considering the facts for both addition and division, it appears that, subject to discounts for the inequalities of the groups in initial ability, there is considerable advantage in the short

period lengths, when the length is two minutes. The advantage there is noteworthy, since in addition the gain is greater than in the longer-period groups, even when their ability was greater in the longer period (Group II); and since in division the gain is so very much greater than in the 20 or 10-minute group.

The facts can be freed from the influence of inequalities in ability at the beginning of practice by comparing only those of equal initial ability. For example, we find in the case of division that those of initial ability 15 averaged in gain 18.0, 26.3, and 23 according as they had practiced in 20-, 10-, or 2-minute periods.

Making such calculations for those of each initial ability in division from 5 to 64 and allowing equal weight to each successive set of five successive groups, it appears that on the average the 20-minute, 10-minute and 2-minute period varieties of practice brought to those of equal initial ability gains in the relation of 100, 110½, and 177.

In the case of addition the same procedure, carried out with those on initial abilities 5, 6, 7, and so on up through 49, gives the following results: according as the practice was in 22½, 15, 6, or 2 minute divisions, it brought to those of equal initial ability gains in the relation of 100, 121, 101, and 146½.

It appears, then, that the superiority of the shortest practice period length remains when inequalities of initial ability are eliminated. It appears further that the periods of intermediate length have really a greater superiority over the longest period than the results irrespective of difference in initial ability showed. There is a positive relation between initial ability and gross gain. Consequently Group III in addition and Group II in division, which happened to be groups of low initial ability, suffered in the comparison.

Dr. Kirby cautions the reader against the conclusion that the short practice periods are wholly responsible for the difference in gain. He points out that those

having the shortest practice periods had more days in which to profit from the practice and in which to profit from the regular school work, that they had a better opportunity of catching the spirit of the experiment, and that consequently they had a more powerful incentive to practice outside. Regardless of the causes, his material shows indisputably that the short periods pay the best dividends. We are therefore unable to subscribe to Dr. Kirby's conclusion that perhaps for administrative reasons it would be better to use the long periods in school. We see no especial administrative difficulties, certainly none that are insuperable, — in having daily drills of two minutes. This is a case, so it seems to us, where an administrative adjustment should be subordinated to a teaching adjustment.

Tests given by Dr. Kirby several months after the final test of the experiment was given, showed a relatively high degree of permanence of the practice effect.

Brown's Experiment on the Value of Daily Drill[1]

HOW THE INVESTIGATION WAS CONDUCTED

Tests were given in the sixth grades of three different public school systems and in the sixth grade of a large private school. The total number of cases recorded in

[1] Used with the permission of Brown and Coffman, authors of *How to Teach Arithmetic*, Ross Peterson & Co.

this study was 222; of these, 110 were boys and 112 were girls.

The three public schools examined are in the Central West. City C has a population of seven thousand; City M, of twelve thousand; and City D, of thirty thousand. The private school is in New York City.

The effects of the drill in fundamentals were shown by a comparison of sections subjected to the drill with sections of equal size and approximately equal ability not subjected to the drill but otherwise undergoing the same arithmetical instruction. The object was to determine the improvement made by the drill class upon its previous record and the improvement made by the non-drill class upon its previous record.

In a given class the tests were conducted at the same hour of the school day, in order to eliminate the time factor as far as possible.

Immediately after the first test was given in each school, half of the classes examined in each city were given five minutes' drill each day upon the fundamental operations in arithmetic,— addition, subtraction, multiplication, and division. The first five minutes of the recitation period in arithmetic were devoted to the drill work. The drill was partly oral and partly written, and the time was about evenly distributed among the four operations.

No special instructions were given to the teachers in charge of the drill sections, except that they were to em-

phasize both speed and accuracy in the four operations, and were to cover the same daily assignments in the textbooks as the class that had no drill. The teachers of the non-drill classes were asked to give no formal drill upon any of the four fundamental operations during the time that this investigation was in progress. These instructions were carefully observed by the teachers.

The drill classes in each city were able to cover the same subject-matter of the text as the non-drill classes of that city. No special tests were given to determine the comparative excellence of the textbook work, but in every case the teacher in charge of a drill class reported that five minutes devoted to drill at the beginning of each recitation seemed to act as a mental tonic. It seemed to energize the pupils and to make them keen for the textbook work that was to follow. All teachers of drill classes reported an improvement in textbook work.

Formal drill work on the four fundamental operations had not been given prior to this investigation in any of the sixth grades examined. Whatever marked changes occurred in all of the drill sections that did not occur in the non-drill sections may reasonably be attributed to the results of the special drill.

RESULTS OF THE DRILL

If the number of problems worked in each test may be taken as a measure of the speed of the pupils, the drill

class increased its speed by 16.9 per cent and the non-drill class by 6.4 per cent.

Since practically all of the pupils finished at least the first six problems in each test, a comparison of the records made on these six problems will give a basis for determining the relative accuracy. Measured by this standard, the drill class made a gain of 11.7 per cent in accuracy, whereas the non-drill class actually lost in accuracy (−1.8 per cent).

The largest gain made by the drill class was in division, 34.2 per cent, which was more than twice the gain made in division by the non-drill class, 15.4 per cent.

If we compare the gain made by the drill class upon its own record with the gain made by the non-drill class upon its own record, we find that the following results were attained:

Drill class gained 2.64 times as much as non-drill class on problems worked.

Drill class gained 2.72 times as much as non-drill class in addition.

Drill class gained 2.68 times as much as non-drill class in subtraction.

Drill class gained 2.21 times as much as non-drill class in multiplication.

Drill class gained 3.13 times as much as non-drill class in division.

Drill class gained 2.57 times as much as non-drill class in total number of points.

The drill classes made from two and one fifth to three and one tenth times as much improvement as the non-drill classes. It is worthy of note that the average age in the drill classes was exactly the same as the average age in the non-drill classes, being twelve and two tenths years in each case.

In the following table the first test was given before the drill was begun, the second test was given immediately after the thirty days' drill, and the third test was given on the first day of the fall term, after a vacation of twelve weeks:

COMPARISON OF THE RESULTS OF THE THIRD TEST WITH THE FIRST AND SECOND

("I" indicates combined drill sections; "II," the non-drill sections.)

I did 26.4 per cent better than on first test and 4.1 per cent better than on second test in number of problems worked.

II did 9.8 per cent better than on first test and same as on second test in number of problems worked.

I did 25.4 per cent better than on first test and 6 per cent poorer than on second test in addition.

II did 7.7 per cent better than on first test and 3.7 per cent poorer than on second test in addition.

I did 46.2 per cent better than on first test and 6.7 per cent better than on second test in subtraction.

II did 20.4 per cent better than on first test and 6.4 per cent better than on second test in subtraction.

I did 31.3 per cent better than on first test and 1.5 per cent better than on second test in multiplication.

II did 11.1 per cent better than on first test and 2.2 per cent poorer than on second test in multiplication.

I did 36.7 per cent better than on first test and 7.3 per cent better than on second test in division.

II did 11.1 per cent better than on first test and 2.8 per cent poorer than on second test in division.

I did 31.7 per cent better than on first test and 0.2 per cent poorer than on second test in total points.

II did 12.16 per cent better than on first test and 2.29 per cent poorer than on second test in total points.

I did 5.2 per cent better than on first test and 0.6 per cent poorer than on second test in first six problems.

II did 3.7 per cent poorer than on first test and 1.3 per cent poorer than on second test in first six problems.

The results of the third test indicated that the superiority of the drill class was maintained over the vacation period. The "period of hibernation" served to increase the speed, while those who had not had the advantage of the drill worked no faster than on the second test. The non-drill section either made no improvement or did worse than on the second test in everything except subtraction.

No investigation has yet been made to determine the relative efficiency of drill periods from one to ten or fifteen minutes, or whether the same length of period is best for each of the fundamental operations.

The Ability of Children to Express Mathematical Judgments

Dr. Frederick G. Bonser[1] of the Teachers College of Columbia University made an investigation of the

[1] Frederick G. Bonser, "The Reasoning Ability of Children." *Teachers College Publications*. No. 37. 1910.

reasoning abilities of children. He tested 757 children in the fourth, fifth, and sixth grades in Passaic, New Jersey.

He tested the mathematical judgment by giving the children twenty questions in arithmetic involving three steps: "First, the analysis of the situation by which the essential features of the problems are conceived and abstracted; second, the recall of an appropriate principle to be applied to the abstracted problem, a search among various principles which may suggest themselves for the right one; and third, involving the second, the inference, the recognition of identity between the known principle and the new situation. Clearly these are examples of deductive reasoning of the usual scientific, involving data, principles, and inferences." The tests are as follows:

Tests I and II

I. *A*. Get the answers to these problems as quickly as you can.

1. If $\frac{3}{4}$ of a gallon of oil costs 9 cents, what will 7 gallons cost?

2. John sold 4 sheep for \$5 each. He kept $\frac{1}{2}$ of the money and with the other $\frac{1}{2}$ he bought lambs at \$2 each. How many did he buy?

3. A pint of water weighs a pound. What does a gallon weigh?

4. At $12\frac{1}{2}$ cents each, how many more will 6 tablets cost than 10 pens at 5 cents each?

5. At 15 cents a yard, how much will 7 feet of cloth cost?

B.

1. A man whose salary is $20 a week spends $ 14 a week. In how many weeks can he save $ 300?

2. How many pencils can you buy for 50 cents at the rate of 2 for five cents?

3. A man bought land for $ 100. He sold it for $ 120, gaining $ 5 an acre. How many acres were there?

4. A man spent $\frac{2}{3}$ of his money and had $ 8 left. How much had he at first?

5. The uniforms for a baseball nine cost $ 2.50 each. The shoes cost $ 2 a pair. What was the total cost of uniforms and shoes for the nine?

II. *A.*

1. 32 plus what number equals 36?

2. If John had 15 cents more than he spent to-day, he would have 40 cents. How much did he spend to-day?

3. What number minus 7 equals 23?

4. If James had 4 times as much money as George, he would have $ 16. How much money has George?

5. What number added to 16 gives a number 4 less than 27?

B.

1. What number subtracted 12 times from 30 will leave a remainder of 6?

2. If a train travels half a mile in a minute, what is its rate per hour?

3. What number minus 16 equals 20?

4. What number doubled equals 2 times three? 3?

5. If 7 multiplied by some number equals 63, what is the number?

"The results of tests I and II were combined, a single quantity thus representing the summarized valuation of each child's mathematical judgment. . . . Below are the tables and summaries of results for grade, age, and sex differences."

FREQUENCY OF ABILITIES BY GRADES

ABILITY	GRADE 4A		5B		5A		6B		6A	
	B	G	B	G	B	G	B	G	B	G
2	1	1		2						
3	1	3		1						
4	4	6	2	2				1		
5	1	3		2				1		1
6	6	4	1	4		2		1	1	1
7		5	1			2	1			
8	8	4	1	3		1	1			
9	1	6				1		1		
10	7	5	1	5	2	3	1	1		2
11	4	3		3	2	1		1		1
12	7	11	4	9	2	4	3	1	1	1
13	2	3	1	2	1	2	1	1		
14	4	4	6	3	3	5	2	3		3
15	1	5	1	3	3			2		1
16	4	7	7	6	2	2	2	6		2
17	2	1	1	3	2	2	1	2		
18	8	2	1	2	2	3	1	3		2
19	3			5		1	5	1	1	
20	6	4	8	9	4	2	4	5	2	3
21		2	1	2	2	4	1			
22	2	2	9	5	5	3	6	8	1	5
23	6	1	1	2	3	2	3	2	1	
24	6	3	11	3	3	2	6	6	4	1
25	1		3		2	1	4	1		
26	3	1	8	4	5	3	11	4	7	7
27		2	1		1		1		1	3
28	1		1	1	6	3	11	6	6	
29	1			1	3	2	1		2	6
30			4	1	2	2	4	8	2	1
31						1	2	1	1	3
32	1		4	1	4	2	10	5	6	8

The meaning of the foregoing table becomes clear when read thus: In grade 4a, one boy had ability 2, one boy had ability 3, four boys had ability 4, and so on down to the case of one boy with ability 32.

In order to see more clearly the differences in results attained by boys and girls in the different grades, the following summary has been made:

MEDIAN ABILITY AND VARIABILITY FOR EACH GRADE

GRADES	M's		Q's	
	Boys	Girls	Boys	Girls
4A	14.50	11.36	5.39	4.21
5B	21.39	15.66	4.75	4.71
5A	22.83	19.00	5.58	6.46
6B	25.63	24.08	4.95	6.42
6A	28.00	25.92	3.96	5.95

The table above should be read thus: The median ability of 4a boys was 14.50, 4a girls 11.36. The ability of half of the boys was within 5.39 of the median for the boys. The ability of half of the girls was within 4.21 of the median for the girls. Q is used here as a measure of variability. "It is gotten by counting in from the low end of the distribution until 25 per cent of the cases are covered; and likewise from the high end of the distribution until the point marking 75 per cent of the cases is reached. These two values give the limit within which exactly 50 per cent of the cases lie. Subtracting the

lower value from the higher and dividing the result by 2 gives the variability in steps of the unit of measure used."

It should be noted that the median performance of boys and girls in response to this test of mathematical judgment steadily increases from the fourth grade to the sixth grade. The supervisor here has evidence sufficient to justify a clear differentiation in regard to the standard of mathematical judgment which should be set up for the fourth grade, the fifth grade, or the sixth grade.

The difference in ability of boys and girls in each grade is also worthy of note. While it is commonly stated that boys exceed girls in the ability to express a mathematical judgment, there is little scientific evidence to support this contention.

The following summary by Dr. Bonser states the case very clearly:

> Other than the evident differences in the foregoing, the simplest summary of sex differences is in a statement of the per cents of the boys who reach or exceed the ability reached by 50 per cent of the girls as given below:

Grade	4A	5B	5A	6B	6A
Per Cent . . .	61.8	71.3	70.7	60.8	65.1

Differences in ability at different ages for boys and girls are also evident, as is indicated by the following table from Dr. Bonser:

FREQUENCY OF ABILITIES BY AGES

ABILITY	AGE 8 B	10 G	10 B	11 G	11 B	12 G	12 B	13 G	13 B	14 G	14 B	16 G
2		1		1		1	1					
3				2			1	1		1		
4	1	4	1		1	1	1	4	1		1	
5				3	1	1		1		2		
6	2	3	3	3		2	1	2	1	1	1	1
7		1		2		3	2	1				
8	3.	1	3	1	1	2		3	3	1		
9		3		2	1	1		1		1		
10	2	2		5	5	5		1	3	1	1	2
11	1		1	2	2	3		2	1		1	2
12 *		5	5	5	5	5	5	5	2	4		2
13				3	2	1	2	2	1	1		
14		4	4	3	3	6	3	3	5			2
15			1	4	3	4	1	2				1
16		3	5	7	6	8	2	4	2	1		1
17		2	1	2			5	1		2		
18	2		4	1	2	7	4	2	1	1		1
19		1	3	1	4	1	1	1	1	2		1
20	2	1	8	6	4	2	7	9	3	5	1	
21		1	1	2	2	2		2	1	1		
22			5	5	6	6	10	8	1	3	1	
23	1		2	4	6	2	2	1	3			
24	2	2	7	3	8	3	6	3	2	3	5	1
25			2		2	1	1	1	3		1	
26	2	4	7	3	7	3	8	7	4	2	4	
27		1	1		3	1		1		1		
28		1	5	3	3	3	11	1	4	2	2	
29	1				2	6	3	1		1	1	
30	3		1	2	7	2	1	5		2		
31			1		1	1	2	2		2		
32	2		4	5	4	2	8	8	5	2	2	
33			1		2	2	1	1		1	1	
34			2		1	2	3	1	2	2	1	
35					1		2		1			
36	2	1	2	1	5	1	1	4	2	1		
37		1	1		1			1				1
38	1		2			2	2	1	1		1	1
39												
40			1			1						
Cases	27	43	84	81	101	93	96	93	53	46	24	16

After calculating the median and the ages for each sex, Mr. Bonser combined them into the following table:

MEDIAN ABILITY AND VARIABILITY FOR EACH SEX

Age	M's		Q's	
	Boys	Girls	Boys	Girls
8 to 11	21.50	15.20	6.05	6.21
13 to 16	23.50	19.00	6.98	7.33

In conclusion Dr. Bonser says:

"A study of the median abilities of the respective grades shows progress through these from grade to grade for both boys and girls. . . . The greatest gain in per cent from one grade to the next for both is from 4A to 5B. The smallest gain is, for boys, from grade 5B to 5A; for girls, from 6B to 6A. . . . In these tests, variability diminishes from grade 4A upward."

In view of the experiments which other investigators have made, it is likely that the reason that variability diminishes throughout the upper grades is that the children in the upper grades are more highly selected. The upper-grade students are the students who have survived the rigors of the elementary school curriculum. The upper-grade work is also probably more nearly standardized.

Concerning age differences, Dr. Bonser says:

"From the array of median abilities of half years, the regularity of progression found on the basis of school grades is not at all in evidence.

"A rhythm in ability is fairly apparent for the boys with its first crest at about 9 years 6 months, the second at about 12 years, and the third at about 14 years 6 months, each crest a little higher than the preceding. . . . Retardation seems evident in the pupils of each respective grade who are from two to four years older than the median age for that grade. There also seems evident another type of retardation in these special abilities, perhaps quite as important, in those pupils who are from two to three years younger than the median age for their respective grades. . . . In tests in which progress from grade to grade and year to year is so very evident, in large groups as here shown, these wide divergencies in ability of lowest and highest quarters of these respective groups indicate that native ability is measured by the tests quite as much as school training."

In regard to sex differences, Dr. Bonser says:

"The one marked sex difference is that of the superiority of the boys in these tests. By every distribution, in every one of its respective divisions, the boys are shown to be more able than the girls excepting in two cases of selected groups, one from grade 4A, and the other from grade 6A. In the youngest 25 per cent of the former grade, the girls just equal the boys in median ability, while for the corresponding group for the latter grade the girls slightly excel. . . . The sex difference diminishes as we proceed up the grades from 4 to 6. Should we proceed far enough (test children in higher grades), we might reach the condition found by Fox and Thorndike in a study of 28 boys and 49 girls of high school age where 'girls do about 5 per cent better on the whole than boys.'"

The percentage of all of the boys reaching or exceeding the ability reached by 50 per cent of all of the boys taken together is 71.43. The median ability of all of the boys taken together is 22.60 with a coefficient of variability of .26; of the girls, is 17.75 with a coefficient of .39.

An Investigation in Regard to the Development of Standard Tests

Following the report of Dr. Stone, Mr. S. A. Courtis, now Director of Research in the Detroit public school, became interested in the development of a series of standard tests for the different grades and different abilities in arithmetic. As a result of his activities, the same arithmetic tests have now been given under as nearly uniform conditions as possible to thousands and thousands of children distributed throughout the United States. 5000 children were tested in Detroit; 33,000 in New York; 20,000 in Boston. The tests have been given quite generally throughout smaller cities of the United States, as Mr. Courtis has succeeded in securing the coöperation of a large number of school supervisors throughout the country. By this means he was enabled to arrive at certain tentative standards of excellence to be attained by children in the different grades in certain abilities in arithmetic. For example, despite the variability in performance of children in the fifth grade in addition, fifth-grade children tended to attain a certain standard of excellence, which was somewhat less than the standard of excellence attained in the same test by children in the sixth grade; which was, in a similar way, somewhat lower than that attained in the same test by children in the seventh grade, and so on. Thus Mr. Courtis has attempted to deter-

mine the standards of attainment which we have a right to expect from normal children in the different grades in addition, subtraction, multiplication, division, copying figures, one-step problems in reasoning, abstract examples in the four fundamentals, and two-step problems in reasoning.[1]

For those who believe in the teaching of the formal side of arithmetic, these tests give a view of the completeness and balance of training afforded by any particular school as compared with the average achievements of schools in other places, and they furnish the teacher with accurate measures of the peculiarities and weaknesses of individuals which enable him to adjust his work accordingly. Three editions of this series, equal in value but differing in every figure, have been published, so that repeated tests may be made either during one year or in successive years. When this is done, the results obtained in one grade may be passed along with the child for the guidance of teachers in the higher grades, the curves for different years being drawn upon the same graph sheet.

It should be said that results secured by tests given under conditions unlike those described by Mr. Courtis [2]

[1] The series is known as series A. A later series of Courtis tests is known as series B.

[2] These tests have been prepared upon regular form sheets and can be secured together with the instructions for giving them from Mr. S. A. Courtis, 82 Eliot Street, Detroit, Michigan.

or using the combinations in a different order cannot be compared with those obtained by Mr. Courtis. The conditions must be duplicated in every respect if the results are to be compared.

These (series A) tests have been given by Mr. Courtis himself, or by teachers under his immediate supervision, to more than 60,000 children in grades ranging from the third to the eighth. As a result of this, standard scores have been computed for each of the grades above the second. The following table presents these scores:

REVISED STANDARD SCORES FOR SERIES A

JUNE, 1913

TEST NO.	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6		No. 7		No. 8	
						Ats.	Rts.	Ats.	Rts.	Ats.	Rts.
Grade 3 .	26	19	16	16	63	2.5	1.5	5.0	1.7	2.5	0.5
Grade 4 .	34	25	23	23	75	3.5	1.8	7.0	3.5	2.9	0.7
Grade 5 .	42	31	30	30	84	4.2	2.6	9.0	5.2	3.1	1.0
Grade 6 .	50	38	37	37	92	4.9	3.5	11.0	6.7	3.4	1.4
Grade 7 .	58	44	41	44	100	5.6	4.5	12.5	8.2	3.7	1.9
Grade 8 .	63	49	45	49	108	6.4	5.7	14.0	9.4	4.0	2.5

This table should be interpreted as follows:

The average third-grade child should do 26 additions in one minute; 19 subtractions; 16 multiplications; 16 divisions; should copy 63 figures; should attempt 2.5 of the speed reasoning problems and get 1.5 right, etc. The attainment of each grade is also indicated by the table. For example: An average fourth-grade child

should do 8 more additions than an average third-grade child. A fifth-grade child should do 8 more than a fourth-grade child. Similar standards are shown for each of the grades. By the use of such standard scores as these, teachers can determine whether each pupil has attained the standard degree of ability for his grade and he can also determine what pupils may be excused from taking work in those phases of the subject in which they have already advanced beyond the standard set for their grade.

Mr. Courtis has prepared a graph sheet by means of which one may graphically represent his attainment in each of these abilities or by which the attainment of any given class or room or school may be represented.

Later Mr. Courtis, feeling the need of a simpler series of tests which would be less expensive in time and money, proposed his series B. In announcing this series Mr. Courtis says, "The experimental work of the past few years proves that if school work is to be made more efficient, there must be:

"1. Definition of Aim.

Illustration: Eighth-grade teachers should practice their children in addition until they can add correctly in eight minutes, twelve examples, each example three columns wide, each column nine figures long.

"2. Limitation of Training.

Illustration: As soon as an eighth-grade child's scores reach the standard, he should be excused from further work in addition, whether he makes a year's gain in one month or ten.

"3. Specialization of Training.

Illustration: The child that is up to standard in addition, but below in subtraction, should be given increased drill in subtraction, the extra time coming from the limitation of training in addition.

"4. Diagnosis and Remedy of Individual Defects.

Illustration: If a child's scores do not rise with class practice, he should be studied individually, his symptoms observed, his difficulties discovered, and the proper adjustment of his work made."

The following median scores for series B are submitted. It should be noted that these are tentative only, and subject to such modifications as may be suggested by additional data:

MEDIAN SCORES: SERIES B

FEBRUARY, 1914, TABULATION

Test 1 — Addition

Source of Scores	Attempts				Rights			
	Detroit	Boston	General	Probable June Standard	Detroit	Boston	General	Probable June Standard
Number in Groups	1,315	20,441	3,618		1,315	20,441	3,618	
Grade 3			3.6	4.6			0.7	2.0
4	5.4	5.3	4.7	6.0	2.7	2.6	1.9	3.0
5	6.7	7.2	7.1	7.5	3.9	3.7	3.9	4.0
6	8.4	8.3	8.0	9.0	4.6	4.9	4.4	5.0
7	9.2	9.2	8.9	10.5	5.4	5.6	4.7	6.5
8	10.2	11.0	9.7	12.0	6.7	7.8	5.6	8.0

Test 2 — Subtraction

Grade 3			3.8	4.0			0.9	1.0
4	5.6	5.5	5.7	6.0	3.1	2.5	1.2	3.0
5	8.0	7.6	6.5	8.0	5.5	4.9	4.5	5.5
6	8.8	9.0	8.9	10.0	6.2	6.3	6.1	7.0
7	9.8	10.0	10.2	11.5	7.3	7.3	6.9	8.5
8	12.3	11.4	11.7	12.5	9.5	8.6	8.4	10.0

Test 3 — Multiplication

Grade 4	3.6	3.9	3.9	4.5	1.0	1.3	1.3	1.5
5	6.4	5.8	6.0	7.0	3.8	3.3	2.6	4.0
6	7.4	6.9	7.2	8.5	4.8	4.8	4.5	5.5
7	9.6	8.0	8.4	10.0	6.0	5.1	5.2	6.5
8	10.5	9.5	9.9	11.5	7.5	6.5	6.4	8.0

Test 4 — Division

Grade 4	1.9	2.6	3.1	3.5	0.7	0.7	0.7	1.0
5	4.9	4.5	4.5	5.0	2.7	2.0	2.3	3.0
6	6.4	5.8	5.8	6.5	4.4	3.3	4.3	5.0
7	8.6	6.9	7.6	8.5	7.1	5.1	5.8	7.0
8	10.3	8.8	9.2	10.5	8.8	6.9	6.3	9.0

Later Standards

After giving almost half a million series B tests to children in forty-two different states, Mr. Courtis found the following to be the approximate median score for June. These standards, or medians, are for speed only, as he did not include problems inaccurately solved.

STANDARD (TIME) SCORE SERIES B: TESTS

JUNE STANDARD INDIVIDUAL SCORE IN THE 4 OPERATIONS WITH WHOLE NUMBERS

Grade	Test 1 Addition	Test 2 Subtraction	Test 3 Multiplication	Test 4 Division
3	3	4	3	2
4	5	6	5	4
5	7	8	7	6
6	9	10	9	8
7	11	11	10	10
8	12	12	11	11
Time allowance, Minutes,	8	4	6	8
	345 487 631 205 943 683 859 175 794	3479127468 1867396737	4179 36	67)61707

"Translated into words the table means that in June the graduate of a grammar school should be able to work correctly in eight minutes twelve examples like that under Test I; in four minutes twelve examples like that under Test 2, etc."

Mr. Courtis says: "The scores given in Table I represent approximately the median speed of work for the different grades and are based upon returns that are nearly nation-wide in scope. The range of variation in

schools in different cities and states is approximately four examples above and below the median; *i.e.*, in some schools the median eighth-grade scores will rise as high as 16 examples in addition and others go as low as 8 examples. Not more than five eighth-grade classes per hundred will exceed these limits, except as very peculiar and special conditions prevail. On the other hand, the range of speed of work in individuals varies from a score of but two or three examples to scores of twenty-four examples, the limit of the test."

Supervision Important

The experimentation of Rice, Stone, and Courtis all suggest the fact that the most important single factor in effective arithmetical instruction is that of close supervision. Mr. Rice was convinced that the most important single factor was that of tests being given by the supervisor. The preliminary investigation of Mr. Stone and the extended investigation of Mr. Courtis are such as to enable a supervisor at the present time to coöperate with other supervisors in the matter of arriving at standards of excellence which we have a right to expect of children in addition, subtraction, etc., in the different grades. No supervisor at the present time can afford to neglect the opportunity of securing the coöperation of others in regard to this important phase of work.

The investigations which have been made in the past confirm the thesis that the supervisor who knows the results to be expected from grade to grade in arithmetic, and who definitely tests the progress of the teachers in attaining these results, will be able to arrive at a satisfactory standard of efficiency in the teaching of arithmetic under supervision.

Mr. Courtis's tests are effective agencies in determining the station of an individual student, a class, or a school. By means of them any student can discover his relative strengths and weaknesses in the abilities tested, and any teacher or supervisor can determine those particular phases of arithmetic that the children should receive additional instruction in as well as those that may for the time being be neglected or dropped altogether. Certainly there can be no justification for continuing practice upon those things that the pupils have already acquired greater skill in than the world needs.

A superintendent can also use such standards as these for estimating the efficiency of his teachers. The application of the standards will not reveal to him the causes of weakness in his teachers, but will reveal the points at which weakness occurs. The superintendent and teacher are then free to try new devices at these points for the purpose of securing greater efficiency.

Unlike some of our present-day reformers who are

wedded to the doctrine of individual indifferences and who advocate a course of study for each child, Mr. Courtis contends that there are certain habits, skills, and knowledges which should be acquired by all the children. Put in his epigrammatic way, he says that instead of giving uniform material in a uniform way and getting varying results (as we have in the past) we should give varying material to varying children to get uniform results. This, we believe, to be an end devoutly to be desired. It calls for a study of the individual capacities and abilities of the pupils and for an intelligent application of materials to suit their needs so that they all attain that uniform standard, — a standard determined by the social serviceableness of the material in the ordinary walks of life.

APPENDIX A

Table Showing Variations in Time Schedule

Akron, Ohio	offers	25	min.	less	in	grades	5 and 6	than	in	4, 7 & 8
Albia, Iowa	"	25	"	"	"	"	7 " 8	"	"	6
Altoona, Pa.	"	50	"	"	"	"	8	"	"	7
Ann Arbor, Mich.	"	15	"	"	"	"	6	"	"	5
Ann Arbor, Mich.	"	30	"	"	"	"	6	"	"	7
Atlanta, Ga.	"	50	"	"	"	"	7	"	"	6 & 8
Berkeley, Cal.	"	20	"	"	"	"	5, 6, 7 & 8	"	"	4
Birmingham, Ala.	"	50	"	"	"	"	7	"	"	6
Boston, Mass.	"	40	"	"	"	"	6	"	"	5
Boston, Mass.	"	20	"	"	"	"	7 & 8	"	"	5
Brockton, Mass.	"	60	"	"	"	"	8 & 9	"	"	7
Chicago, Ill.	"	100	"	"	"	"	5, 6 & 7	"	"	4
Chicago, Ill.	"	145	"	"	"	"	8	"	"	4
Cleveland, Ohio	"	15	"	"	"	"	5	"	"	4
Cleveland, Ohio	"	20	"	"	"	"	7	"	"	6
Danbury, Conn.	"	25	"	"	"	"	7 & 8	"	"	6
Davenport, Iowa	"	25	"	"	"	"	6, 7 & 8	"	"	4
Davenport, Iowa	"	50	"	"	"	"	5	"	"	4
East Saginaw, Mich.	"	30	"	"	"	"	8	"	"	7
Emporia, Kan.	"	15	"	"	"	"	8	"	"	7
Dover, N. H.	"	125	"	"	"	"	8	"	"	7
Dunkirk, N. Y.	"	75	"	"	"	"	7 & 8	"	"	6
Gloversville, N. Y.	"	25	"	"	"	"	6	"	"	4
Gloversville, N. Y.	'	50	"	"	"	"	5	"	"	4
Joliet, Ill.	"	30	"	"	"	"	7 & 8	"	"	6
Kalamazoo, Mich.	"	20	"	"	"	"	7 & 8	"	"	6
Keokuk, Iowa	"	33	"	"	"	"	4	"	"	3
Keokuk, Iowa	"	38	"	"	"	"	5	"	"	3
Keokuk, Iowa	"	21	"	"	"	"	6	"	"	3
Keokuk, Iowa	"	17	"	"	"	"	7	"	"	3
Los Angeles, Cal.	"	10	"	"	"	"	7 & 8	"	"	6
Manchester, N. H.	"	50	"	"	"	"	6 & 7	"	"	5
Memphis, Tenn.	"	10	"	"	"	"	8	"	"	7
Minneapolis, Minn.	"	45	"	"	"	"	7 & 8	"	"	6
Muscatine, Iowa	"	45	"	"	"	"	7	"	"	6
Newton, Mass.	"	30	"	"	"	"	7 & 8	"	"	6
Niles, Ohio	"	100	"	"	"	"	8	"	"	7
Oakland, Cal.	"	120	"	"	"	"	7 & 8	"	"	6

Olean, N. Y.	offers	25	min.	less	in	grade	8	than	in	7
Plainfield, N. J.	”	25	”	”	”	”	4	”	”	3
Plainfield, N. J.	”	125	”	”	”	”	6	”	”	5 & 4
Plainfield, N. J.	”	150	”	”	”	”	7	”	”	5 & 4
Portland, Maine	”	100	”	”	”	”	8	”	”	5 & 4
Portland, Maine	”	50	”	”	”	”	7 & 8	”	”	6
Riverside, Cal.	”	5	”	”	”	”	7 & 8	”	”	6
St. Cloud, Minn.	”	25	”	”	”	”	6 & 7	”	”	5
St. Cloud, Minn.	”	20	”	”	”	”	8	”	”	5
Savannah, Ga.	”	50	”	”	”	”	6	”	”	5
Savannah, Ga.	”	100	”	”	”	”	7	”	”	5
Savannah, Ga.	”	75	”	”	”	”	8	”	”	5
Schenectady, N. Y.	”	25	”	”	”	”	5 & 6	”	”	4
Schenectady, N. Y.	”	70	”	”	”	”	7	”	”	4
Schenectady, N. Y.	”	100	”	”	”	”	8	”	”	4
Spokane, Wash.	”	15	”	”	”	”	7	”	”	6
Spokane, Wash.	”	10	”	”	”	”	8	”	”	6
Springfield, Mass.	”	25	”	”	”	”	5, 6, 7 & 8	”	”	4
Tacoma, Wash.	”	25	”	”	”	”	8	”	”	7
Watertown, N. Y.	”	50	”	”	”	”	6	”	”	5 & 7

Total cities	40
Total cases	59
Cases of reduction in grades 7 and 8	40

INDEX

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